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SOME RECENT PHYSICAL THEORY.[1]

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The announcement by Wilhelm von Röntgen in December, 1895, of his discovery of a new kind of radiation, created an immediate and widespread interest which has probably not been exceeded in the history of science. But the importance of that event was far greater than the announcement of a striking and astonishing discovery. The subsequent developments of physics and chemistry show that Röntgen's discovery marks the practical beginning of a new era in physical science. While our knowledge of the nature of X-rays has increased little beyond what we learned in the first few months after their discovery, the investigations started and suggested by Röntgen's discovery have revolutionized our concepts and theories in nearly every field of physics. Thus Sir J. J. Thomson was started with a new impulse into the investigations of the nature of the cathode rays and the mechanism of the electrical conductivity of gases, and these investigations led directly to the discovery of the corpuscle or electron. In France, Becquerel was inspired to study the fluorescent effects of many minerals, and the next year after Röntgen's announcement came the epoch making discovery of the Becquerel rays, a phenomenon which might have remained unknown for another generation, had it not been for the suggestion of the Röntgen rays. Following this lead, we have the great investigations of the Curies, Rutherford, Ramsey, and many others in the subject of radioactivity and of the nature of the chemical elements. The concepts thus introduced and the methods made possible by the new phenomena, have enabled physicists and chemists to re-open old investigations, and the facts thus revealed have given us new interpretations of many old results that were thought complete. We are still in the midst of this scientific era, and much remains speculative and unsettled, yet it has seemed well to spend a short time in discuss-

[1]Read at the meeting of the Central Association of Science and Mathematics held with Northwestern University, Evanston, Ill., Nov. 29, 1912.

ing some of the newer concepts which must affect our beginning statements and definitions, and hence must interest us most vitally as students and teachers of physics.

Perhaps no concept ever dominated science so completely as did the concept of the æther dominate physics during the nineteenth century. The æther was the medium of which eminent thinkers and writers told us that we knew more about than we do about air or any other form of matter. By this medium we explained the action and nature of light, the transmission of electrical and magnetic induction, and even of gravitational force. Matter itself was supposed to be whirls or vortices in this æther, and positive and negative electricity were simply boundary conditions due to the displacements of the æther. Many of the greatest minds of the nineteenth century spent their best energies in the development of a physics of the æther. Such intellectual giants as Fresnel, Kelvin, Maxwell, Stokes, MacCullough, Helmholtz, Hertz and Poincaré worked at the problems of this universal medium which was thus to advance us further in solving the mystery of the universe. The æther was primarily a concept to explain the propagation of such periodic disturbances as visible and invisible light. The concept of a luminous æther was first stated in a tangible form by Christian Huygens and Robert Hooke in the seventeenth century, but it was not until the very beginning of the nineteenth century that it displaced the corpuscular or emission theory which we ascribe, perhaps improperly, to Sir Isaac Newton. Then came the brilliant discoveries of the interference of light by Dr. Thomas Young in England, and Augustin Fresnel in France, with their direct explanation on the undulatory æther hypothesis. This was followed by Fresnel's work in the polarization and double refraction of light, so that before Fresnel's death in 1827, there were few who questioned the undulatory theory of light and the existence of a luminous æther. But there were difficulties in the concept, and it called for all the skill of Kelvin and other great mathematical physicists of the middle of the last century to develop the so-called "elastic solid theory" of the æther, which should meet the facts of with even approximate satisfaction.

During these same years, Michæl Faraday had been developing his theory of electric and magnetic fields, and had given to science his great concept of lines of electric force. Faraday did not at first assume any medium for the lines of force, and even as late as 1851, he seems to have had his doubts about the

necessity of an æther for the transmission of electrical and magnetic forces across space. While he suggested that the luminous æther might "have other uses than simply the conveyance of radiation," and might be the vehicle of magnetic force, it was not Faraday, but James Clerk Maxwell, who developed the concept of the electromagnetic luminous æther. Maxwell took up the problem which was placed before him by Faraday's "Experimental Researches in Electricity." He was equipped for the work by his training in the University of Cambridge which has for centuries been one of the world's great mathematical centers and he developed a theory which has been the admiration of both physicists and mathematicians. Maxwell's electromagnetic theory identified electrical and magnetic phenomena as disturbances in the same medium as light, and indeed made light an electromagnetic wave in the æther. This theory gained recognition slowly, but after 1887 and 1888, when Heinrich Hertz demonstrated by his brilliant experiments the existence and properties of electric waves, there seemed to be nothing more firmly fixed in physics than the existence of an electromagnetic luminous æther. Thus Sir Oliver Lodge in his book on Modern Views of Electricity, edition of 1899, describes the æther as "one continuous substance filling all space; which can vibrate as light; which can be sheared into positive and negative electricity; which in whirls constitutes matter; and which transmits by continuity and not by impact every action and reaction of which matter is capable." A theory which was so dominant with the most advanced and profound thinkers in physical science, naturally came to be the view presented in the manuals of physics, so that we find the æther concept of electricity and light was the view presented in our most progressive elementary text-books of scarcely a decade ago. But while these elaborate and extensive theories of the æther were held generally, there were some thinkers who felt that the experimental foundations were neither broad nor sure enough for such a big structure of theory. Thus Lord Salisbury, in the Presidential address before the British Association for the Advancement of Science in 1894, describes the æther as simply the subject of the verb "to undulate." That is, he calls attention to the fact that the æther is a concept to explain the transmission of light waves, and that is all we really know about it. The rest is speculation.

One of the greatest difficulties of the æther theory, was the

explanation of the nature of the electric charge as it appears in electrolysis. The explanation of the electric current through metallic conductors in the Maxwell theory, was certainly not simple, though it gave a possible solution, but the passage of electricity through an electrolyte was confessedly incomplete. Much more incomplete was the theory of electric discharge across gases. These phenomena, we are told, greatly interested Maxwell, but the whole subject of discharge through gases occupies but a few short sections in his great "Treatise on Electricity and Magnetism." While Faraday, Plücker and others had fixed some of the fundamental facts of discharge in vacuum tubes, yet at the time of the publication of Maxwell's treatise in 1873, the facts known were too fragmentary and indefinite to form the basis for any general theory. At the time of Röntgen's discovery, there had sprung up a renewed activity in the investigation of the phenomena of electric discharges in exhausted tubes. Lenard had discovered rays that penetrated aluminum windows in the tube, and J. J. Thomson had already begun his epoch making investigations in this field. Röntgen's startling discovery gave the new impulse to Thomson's work and this resulted in the atomic or electron theory of electricity. The electron theories that followed have completely displaced the æther theories for many phenomena. Thus the electric charge is no longer regarded as a shear of the æther, but as a collection of electrons; the electric current in a wire becomes a flow of electrons, and not a breaking down of the æther strains along the metallic wire; an elementary or molecular magnet is no longer due to æther whirls, but is due to the rotation of one or more electrons about the material atom. The æther vortex theory of matter of Kelvin and Helmholtz has been replaced by the corpuscular theory of matter of J. J. Thomson. The inertia of matter, thus becomes the electromagnetic inertia of the moving electrons of which all matter is built up. Within a little over a decade, the corpuscular theory developed by Thomson and his school displaced a large part of the physics of the electric æther which the men of the 19th century built up with such great labor. The investigations by Rutherford and the Curies in radioactivity contributed largely to this revolution of our concepts. Rutherford showed that radium gave off three types of rays, and that two of them, the d and b rays, are corpuscular or atomic in nature. The third type of rays, the r rays, it has been commonly thought are the same as the X-rays of Röntgen, and these have been

thought of as pulses of the æther produced by impacts of electrons. But even here, there has arisen a question of the necessity of æther, for a brilliant experimenter Professor Bragg of Leeds, believes that he has shown that the r rays are corpuscular or atomic. If the r rays are corpuscular, then Bragg's conclusion is that X-rays are corpuscular and not æther pulses. This starts another question, for the X-rays and the ultra violet light show many similar properties and so we are led to ask, is not ultra violet light corpuscular as well as the X-rays? And if ultra violet light is corpuscular, why is not all light and radiation to be considered as corpuscular. Bragg's concept of the r rays is that they are doublets of positive and negative corpuscles, so that the periodic character of such a radiation might come easily as a consequence of the vibration of the advancing doublets. If this bold speculation prove to be true, we shall be back to a Newtonian corpuscular theory of light, and the concept of an æther would be rendered still more unnecessary.

But how explain the existence of lines of electric and magnetic force in a vacuum? The æther theory said, "they are made of the æther, the medium which fills all space and through which light is propagated—that is, they are the lines of strain and stress in the intervening medium." The newer school seem to have gone back to Faraday's very first tentative ideas of lines of force, and to give these lines an objective existence independent of any medium. Thus Mr. Norman Campbell in his recent book, entitled "The Principles of Electricity," says: "Lines of force are just lines of force independent for their existence of all surrounding bodies, and there is no more to be said about them. If lines of force passing through sulphur are not made of sulphur, there is no need, when the lines pass through a vacuum to imagine the vacuum filled with a substance of which the lines may be made; in other words, our electrical theory, so far from providing additional support for the conception of the æther filling all space, does not require such a conception at all. All it needs is the conception of lines of force; where there are no lines there is no need for the presence of anything at all. We do not require to imagine present everywhere a substance of which the lines of force may be made when charged bodies come into the neighborhood, for the bodies bring their own lines with them, ready made and unalterable." He says further in emphasis, "The idea that an æther existing everywhere is needed for Faraday's theory is not necessary; all that is necessary are the lines

of force, which are not made of the medium through which they pass." Mr. Campbell, whom we are thus quoting represents views which are more radical in details than some physicists agree to, but certain it is, that the electrical and magnetic æther, even if we call intervening space by that name, is an entirely different conception from that held so generally a dozen years ago or less, and which still persists in text-books.

We turn now to an entirely different line of inquiry—that is the investigation of the radiation from a black body. When a blackened body, such as a carbon filament, is raised in temperature, it gives off radiant energy which increases in amount and also in frequency as the temperature rises. The law of radiation from an ideal "black body," that is, from a body which radiates and absorbs perfectly, has been studied by many physicists and numbers of theoretical formulæ have been proposed. About 1895, a group of German physicists, prominent among whom were Professors Lummer, Pringsheim and Kürlbaum of Berlin, began to give us exact experimental results on the radiation and the temperature in the case of a uniformly heated cavity which was nearly closed, and which for all practical purposes realized Kirchhoff's ideal black body. These and later results have afforded a guide and test to the theoretical radiation formulas of Wien, Rayleigh, Jeans, Planck, Kunz and others. Of these formulas, that of Planck has been most widely accepted, though the firmness of its theory has been questioned and some think the agreement with experiment to be accidental and apparent and not real. Planck started with the concept of the electromagnetic origin of radiant energy. He assumes it due to vibrating electrons in the atom and takes Hertz electric oscillator as the type of the electronic oscillator. This is the common type familiar to students of electric waves and simple wireless telegraphy. As stated the formula thus derived agrees fairly well with our present experimental results for a wide range of temperatiures and hence has received wide acceptance. The most striking fact of Planck's radiation theory is, however, that it leads to Planck's "quantum" or atomic hypothesis of radiant energy. This hypothesis says that radiant energy is not to be considered as infinitely divisible and continuous, but as discrete and made up of a great number of finite and probably equal parts, called by Planck, "quanta." Professor Einstein of Zürich, who is one of the first of living mathematical physicists, has gone beyond Planck's conception, and says that a ray of light consists of

innumerable atoms of energy or light quanta, that is, that the light exists in space in discreet light atoms, or quanta, and is not continuous as the æther wave theory assumes.

It is not possible in a short paper to present the detailed reasons which have led Planck, Einstein and others to these new views of the nature of radiant energy. The papers of Planck, Einstein, Sommerfeld, Nernst and others must be studied by one who wishes to understand how firm a hold this atomic theory of radiant energy and light has upon a large group of the most profound thinkers in physical science today. Certainly if we are to accept an atomic theory of electricity and an electronic theory of matter, then there is nothing strange or absurd in an atomic or corpuscular theory of the light and radiation coming from matter, for Zeeman, Lorentz and others have shown the close connection between the vibrating electron and the emitted light. It would however sound strange to Helmholtz and the physicists of his generation to learn that we have come back to a theory so closely resembling the Newtonian emission theory. We thus see that the electron theories are leading us to ideas of discrete quantities of not only electric but also light energy. This is manifestly not in accord with the concept of a continuous æther.

One of the fundamental questions about the æther has always been, is the æther stationary or does it move with the earth? The experiments on this question are so contradictory that a whole group of leading scientific men have been led to deny not only the existence of the æther, but also to revise the "common sense" ideas of time and space which have always been used by physicists, whatever their metaphysical creed might be. On one hand we have the famous aberration observations of James Bradley made in 1728, that a ray of light from a star appears to come at a slanting angle. This is explained directly as due to the composition of the velocities of the earth and of the light. Thus if the light is a disturbance in the æther, the æther must be stationary in reference to the earth. On the other hand, we have the recent experiments of Michelson and Morley and Miller, of Brace, and of others, which show that the apparent velocity of a ray of light does not depend upon whether the direction of the light is the same as that of the earth or not. The direct interpretation of these last experiments is that there is no relative motion of the æther and of the earth. It is probable that these experiments on æther drift have been with most physicists the greatest reason for questioning the æther concept of luminous

transmission. Thus Fitzgerald and Lorentz have suggested a possible explanation in assuming that material bodies shrink in the direction of the æther drift, and hence the change in the light velocity would be hidden. This is however giving us a meter bar of shifting length, and most of us like to think of something as fixed. A recent writer says, "Almost any experimental result can be reconciled with almost any theory if sufficient subsidiary assumptions are made; the only question is whether it is worth making them." As said above, many do not think the original concept is worth making the necessary subsidiary assumptions. The question involved in a stationary or moving æther is a very big one, and it is mentioned here simply as another evidence of the drift away from the æther concept, in its old form at least.

From what we have been saying it will be seen that there is at present a decided tendency in physics to go back to the older separate entities and to abandon the continuous fluid ideas associated with the æther concept. Thus we have the electron instead of Maxwell's æther "displacement" and if Planck's radiation theory is to be accepted, we have a corpuscular conception of energy. An interesting extension of this same idea, is given in Professor Callendar's address before the Physics Section of the B. A. A. S. in its meeting at Aberdeen last August. Professor Callendar suggests that recent discoveries point towards a material theory of heat, and he then proceeds to show that a modified caloric theory of heat affords reasonable explanations of thermal phenomena. He further advances the speculation that this caloric may be neutral corpuscles. That the countrymen and students of Kelvin, Joule, Tait, Rankine and Tyndal should entertain with scientific seriousness the discussion of a corpuscular theory of heat by one of their leading physicists, is indeed very significant. If, however, an atomic or quanta theory of radiant energy is to be accepted, it is certainly not many steps to a caloric theory of heat.

It is thus evident that we are in a period of new fundamental theories in physics. To the student of physics it is a most interesting and stimulating time, with opportunities and invitations for telling work in nearly every field of physics. To the teacher who is stating and presenting the elements of the subject, the situation is not simple. To keep abreast of the advances in fundamental concepts, and still keep on safe ground is not easy. Further, the theory that appears simplest to present may not be

that which is nearest the truth. Thus Professor Callendar suggests that the kinetic theory of heat has come to be adopted to the exclusion of the material idea, because, quoting his words, "The kinetic theory is generally preferable for elementary exposition." In this particular case most of us are not yet ready to abandon the essentials of a kinetic theory of heat, but the idea suggested of giving a theory because it is "preferable for elementary exposition" raises a question. There are indeed those who hold that a theory is simply scaffolding and not a serious attempt to build a permanent structure. There is a system of philosophy which claims to be copying the methods of natural science which confesses that its explanations are purely speculative and cares nothing for reality. Indeed it is claimed that the number of possible working theories of material phenomena is indefinite and that the theory that we adopt is simply a question of convenience in thinking. The general introduction of such metaphysical ideas into physics would be fatal to advance. As students and teachers of physics, we must believe and teach that a physical theory is a real explanation of real phenomena, if the physics of this century is to equal and excel the triumphs of the physics of the last century.

MID-WINTER MEETING OF THE NATIONAL EDUCATION ASSOCIATION.

The meeting of the Department of Superintendence and such other departments as usually have a mid-winter session will be held at Philadelphia, February 25-28, 1913. The Bellevue-Stratford Hotel will be the headquarters. It is expected, although the formal action by the railroad associations has not yet been taken, that the return limit of the tickets will be extended so that those of our members who desire may take in the inauguration ceremonies at Washington the week following the meeting.

At a meeting of the Executive Committee of the National Education Association, held in Chicago, Wednesday, October 23, it was decided that, provided satisfactory railroad rates and ticket conditions are secured from all the various passenger associations, the next meeting of the National Education Association will be held in Salt Lake City, Utah, July 7-11, 1913.

Yours truly,

D. W. Springer, *Secretary.*

THE USE OF THE ELECTRIC HEATER IN EFFICIENCY TESTS.

By Ernest Reveley Smith,
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We are living in an age of commercialism. The relation of output to input is the great factor which determines our investments whether large or small. What is so general about us cannot fail to enter our laboratories. The toys that have been used so long as equipment are rapidly disappearing, their purposes well served. In their places are coming the newer commercial appliances, the experimental uses of which commend themselves instantly to the boy or girl as something worth while.

Among the commercial offerings to the Physics laboratory, few have greater possibilities than the various types of electric heaters. The very fact that the electric stoves, flatirons, immersion heaters, etc.,[1] are taking their places among the things of our every day life makes the use of them in the laboratory both interesting and profitable.

Also they are the most adaptable of any of the laboratory

Fig. 1.

equipment for work along efficiency lines, since all that is necessary for performing the experiment, besides the heater itself and the sources of current, is common equipment found in every laboratory. Generally we use an electric stove (fig. 1), with a

[1]The appliances used in these tests are furnished by the Pacific Electric Heating Co., Chicago, Ill., or Ontario, California.

voltmeter and an ammeter of suitable ranges, a flat bottom aluminum sauce pan, a watch and a thermometer.

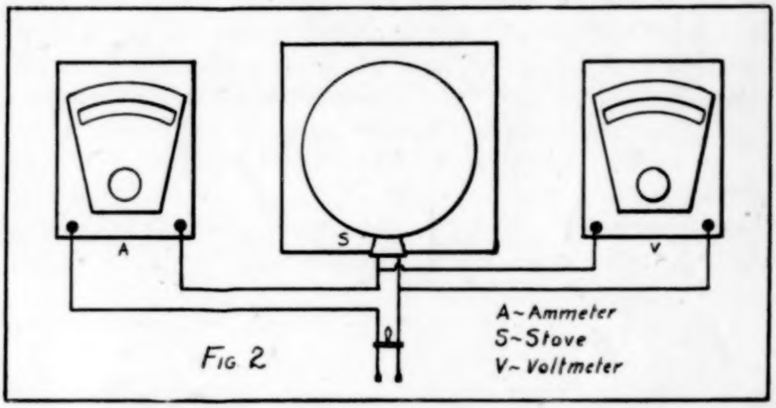

FIG 2

The ammeter is connected in series with the stove and the voltmeter shunted across its terminals, (fig. 2). (A wattmeter may be used in place of these instruments). While the kettle, and the kettle with the water are being weighed, the current is turned on through the stove, so that it may come up to the normal working temperature. In this way very little heat is absorbed by the stove itself during the actual tests.

The temperature of the known weight of water is now taken and the kettle placed on the stove just as the stop watch is started. Voltmeter and ammeter readings are taken every minute and their average readings used, since there is usually considerable variation in the potential of city currents. At the end of a given time (ten minutes), the temperature of the water is read after stirring, and the current is cut off.

From the average current and fall of potential through the stove, its resistance is computed. The heat developed in the stove is computed from the well known formula, calories= $0.24\ C^2\ R\ t$. The water equivalent of the kettle is found from its mass and specific heat. Then the heat absorbed is the mass of the water including the water equivalent of the kettle, multiplied by the change in temperature. The efficiency is now obtained by dividing the calories absorbed by the calories developed.

The efficiency tests that have been made in our school for the past three years have given results varying from 45% to 50% with one stove and from 65% to 70% with another.

This experiment may be varied in several details. The ap-

parent efficiency will be raised from 10% to 15% by using a large amount of water in place of 400 or 500 grams. Covering the kettle will usually raise the results by 3 % or 4 %. Again enclosing the kettle and stove in an asbestos jacket will give a result some 5% to 10% higher. This jacket is easily made from asbestos sheeting. Another variation brings into use the heat of vaporization. The experiment is continued until part of the water has boiled away. The kettle and contents are then weighed. The heat absorbed is equal to the sum of the heat necessary to bring all the water to the boiling point and that required to vaporize the water lost by boiling. This method will give results slightly higher than the first.

The immersion heater (fig. 3) gives much higher results than

Fig. 3.

the stove. Our tests have shown an efficiency varying from 90% to 98%. The heater is tested in the same way as the stove. For general use about a laboratory this device is very satisfactory as it will heat water more quickly than gas and may be used with any kind of a dish.

The flatiron (fig. 4) makes an excellent stove. In fact, many

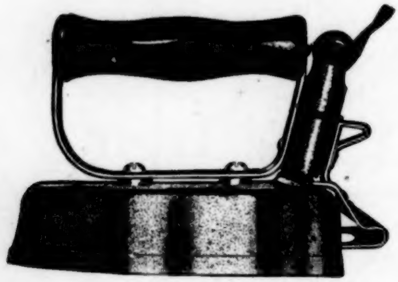

Fig. 4.

manufacturers furnish a stand to hold it inverted as well as a dish shaped to fit its working surface. Its efficiency is not as high as the immersion heater or stove, ranging from 40% to 60%, depending largely upon the shape of the kettle.

In laboratories having electricity but no stove, an incandescent bulb may be used for efficiency tests. If the experiment is per-

formed first with a covered opaque calorimeter, and then with a glass jar, the relative amounts of energy given off as heat and light may also be determined.

In any of the above experiments the cost of electricity may be easily computed. If the pupil has found, earlier in his work, the cost of using a gas stove or burner for a similar length of time, he now has data for an interesting comparison.

Usually I divide the class into several squads of five or six for these experiments. While one squad is performing this experiment, the other members of the class are working on an experiment for which we have individual apparatus. One pupil from each squad weighs the kettle and water, another reads the thermometer, another has charge of the wiring, while others read the voltmeter and ammeter or hold the watch. This insures the constant attention of each member of the squad since he has something to do which is definite and vitally important to the experiment. Of course, the entire experiment may be performed by two pupils, if desirable, or made a class exercise, letting several pupils make the readings for the class. Whichever way it is done, it furnishes one of the most instructive as well as popular experiments in our laboratory.

The direction sheet given our pupils follows:

Efficiency of Electric Heater.

OBJECT. To determine the efficiency of an electric stove.

APPARATUS. Small disc electric stove; voltmeter; ammeter; aluminum kettle; thermometer; balance and weights; stop watch and source of current.

MANIPULATIONS. Weigh the kettle, empty and dry. Fill it about ⅔ full of cold water and weigh it again.

Connect the ammeter in series with the stove and shunt the voltmeter across its terminals.

Close the circuit through the stove and leave the current on for one minute before putting the kettle of water on. During this minute stir the water in the kettle and take its temperature. At the end of the minute put the kettle of water on the stove and start the stop watch.

Take a reading of the voltmeter and the ammeter every minute and use the average reading of each instrument in your computations. Stir the water occasionally.

At the end of 9½ minutes begin to stir the water and ½ minute

later read the temperature of the water in the kettle just as you take it from the stove. Immediately open the circuit.

COMPUTATIONS. From the average current and voltage determine the resistance of the stove. Compute the heat developed in the stove in ten minutes.

Using the specific heat of the aluminum kettle as 0.214, compute the water equivalent of the kettle. Add this to the weight of the water in the kettle, and compute the number of calories of heat absorbed by the water and kettle in ten minutes.

From the heat developed by the current and the heat absorbed by the water and the kettle, determine the efficiency of the stove. Express as a per cent.

Find the cost of the electricity used in the ten minutes, if electricity costs 10 cents per kilowatt-hour in Syracuse.

RESULTS.

Weight of kettle ....gms.
Weight of kettle and water..............................gms.
Weight of water ....gms.
Temperature of cold water...............................°C.
Temperature of water after 10 minutes...................°C.
Change in temperature of water..........................°C.
Fall of potential through stove.........................volts.
Current through stove ....amps.
Resistance of stove.....................................ohms.
Heat developed in stove in 10 minutes...................cal.
Water equivalent of kettle..............................gms.
Heat absorbed by water and kettle in 10 minutes.........cal.
Efficiency of electric stove.............................%
Cost of electricity for the experiment..................cts.

BUREAU OF EDUCATION LIBRARY.

The Commissioner of Education of the United States is trying to make the library of the Bureau of Education a complete reference library on all phases of education. To assist in this he wishes to obtain, as soon as issued, two copies of all reports, catalogs, circulars of information, and all similar publications of State, county, and city departments of education, and of education associations, boards and societies. All persons responsible for the distribution of any such matter are requested to send two copies to the library of the Bureau. If the postage would be considerable, the librarian should be notified by card, when free mailing labels will be sent.

Address all communications to The Librarian, Bureau of Education, Washington, D. C.

A METHOD FOR THE PREPARATION OF EARTHWORMS AND SMALLER OLIGOCHAETES FOR STUDY.

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Any one who has attempted to prepare specimens of earthworms for use in the laboratory realizes how difficult it is to kill and fix the specimens in a perfectly straight condition. This difficulty is due to the tendency of the worms to exhibit a variety of distortions when submitted to the killing and fixing reagents. The use of anaesthetics, particularly chloretone, reduces, to some extent, the danger of distortion, but by no means eliminates it entirely. While engaged in the preparation of earthworm material, the writer devised a method which proved to be simple, convenient, and very successful in securing straight specimens. It is, of course, possible that this method is not a new one and that it may have been used by some other worker, but the writer has no knowledge of any published account of it and since it has been found so very satisfactory, it seems possible that it might be used to good advantage by teachers and others who have occasion to deal with the preparation of oligochaete material.

The apparatus required for the execution of this method is simple and easy to provide. It consists of a plate of glass, two glass trays, a pipette, a hard wood needle, a hard wood spatula, and six wooden strips. Any piece of glass will do provided it be of convenient size, as for example, 22 x 30 cm., and its surface be perfectly plane and smooth. The glass tray requirement can be fulfilled by candy trays, large Petri dishes or similar receptacles which must be large enough to contain an object at least six inches long. The wooden needle referred to is a hard wood stick of size and shape similar to an ordinary pen holder, but which tapers to a point at one end. The wooden spatula may be made separately or may be combined with the wooden needle, one end bearing the needle and the other the spatula. The needle and spatula may be made of other substances, such as glass or horn, but metal instruments should be avoided if corrosive sublimate mixtures are to be used as the fixing agents. The wooden strips should be of some light material, the sides perfectly straight and smooth and at right angles to each other, and each strip must be longer than the longest of the worms to be used. In the middle of the length of each a small hole should be bored. The writer uses six seasoned bass wood strips, 0.4 x 0.6 x

15 centimeters in dimensions and these serve all purposes except in the case of very large worms, when it is necessary to use strips of greater length and thickness. In addition to the above named working materials, the usual reagents, such as chloretone, fixing fluid, and the usual grades of alcohol must be provided.

When the living worms are at hand and are ready to be killed and fixed, the procedure is as follows:

1. Pour the fixing fluid into one of the glass trays to a depth of about two centimeters. Put all of the wooden strips into the tray and allow them to soak for several minutes.

2. Put water to a depth of about two centimeters into the second tray and transfer the worms to it. Add chloretone gradually until the worms are completely anaesthetized. The writer determines this point by placing the needle at one side of the worm and sweeping the animal rather vigorously about in the water; if it responds in any way the anaesthesis is not complete; if the worm is perfectly limp and passive, showing no signs of motion of any sort, the anaesthesis is usually complete. It is very important that one be sure of this point before passing on to the next step.

3. Transfer one of the wooden strips from the fixing fluid to the glass plate by thrusting the end of the wooden needle through the hole in the strip. Carry over as much of the fixing fluid as will cling to the strip. Next transfer an anaesthetized worm from the chloretone solution to the glass plate and by means of the needle bring the worm up parallel to and into contact with the long edge of the wooden strip. Immediately transfer a second wooden strip to the glass plate and bring it up to the other side of the worm. Do not pinch the worm between the strips, but hold just firmly enough to prevent bending in either direction. In case that it should happen that the anterior end tends to bend up from the glass, it can be gently held down with the end of the pipette until fixed. The fixing fluid which was carried over on the strips soon makes a preliminary fixation but in case the quantity carried over happens to be too small, two or three drops can be added by means of the pipette. Now bring over another worm, place it against the other side of the second strip, bring a third strip from the fixing fluid and place against the other side of the worm and proceed as with the first worm. Continue this process until the six strips are all placed parallel on the glass plate, each separated from the others by a

worm. By the time the sixth strip is in place the first worm is usually sufficiently fixed to prevent any distortion. Then—

4. Remove the first strip, dip it into the fixing fluid, and transfer to the front of the array of strips and worms and use it for worm number six. By means of the spatula, transfer the first worm to the glass tray containing the fixing fluid. Continue this procedure until all of the specimens are thus disposed of. Leave specimens in the fixing fluid until fixation is complete. From this point on the procedure depends upon the fixing agent and the ultimate disposal of the material and requires no comment since the methods are well known.

Besides being a successful means of securing straight specimens, the method is a rapid one. On one occasion the writer killed and fixed fifty specimens of a certain species of earthworm in good condition in one and one-fourth hours.

A Method for Some of the Smaller Oligochaeta.

It is obvious that the above described method applies only to the larger forms, the true earthworms, and would not be practical in the case of the smaller species (Enchytraeidae, Tubificidae, etc.). For these forms, particularly the Enchytraeidae, the writer uses a modification of the above described method which has proved to be far more satisfactory than any other method known to him. It is as follows:

1. Apparatus.—Same as used in the above method with the following exceptions: The hard wood needle should be smaller and more acutely pointed, and toothpicks (the square kind) or other small sticks of similar material, size, and shape, should be substituted for the wooden strips.

2. Procedure.—It is possible to get satisfactory results by proceeding as in the method described for earthworms but the writer gets good results by using a somewhat simpler scheme, which is as follows: Transfer a completely anaesthetized worm to the glass plate, straighten it as much as possible by means of the needle, then bring over a saturated toothpick carrying all of the fixing fluid that will cling to it, and place it on the glass plate near and parallel to the worm; then push the toothpick with its adhering fixing fluid up into contact with the worm. In practically every case the surface tension of the fixing fluid is sufficient to hold the worm firmly against the straight surface of the toothpick and thus prevent distortion. A preliminary fixation is soon

accomplished and then the worm can be transferred to the vessel containing the fixing fluid.

This method has been successfully employed for Enchytraeidae measuring only three millimeters in length.

LARGEST PRODUCER OF GLASS SAND.

Pennsylvania produces 30 per cent of the sand used in glassmaking in the United States—about 400,000 tons. The average value of glass sand in Pennsylvania, according to the United States Geological Survey, is $1.40 a ton.

THE EARLIEST KNOWN PAIR OF SPECTACLES.

The facts that the Chinese have long known of spectacles and that snow spectacles have been employed by the Samoyed tribes near the arctic circle have been frequently remarked on in books of travel, and Layard found a plano-convex lens of rock-crystal in the ruins of Nineveh; but that these oriental races knew of the use of eye-glasses before the fifteenth century is a matter of grave doubt. All European references to the use of spectacles before the year 1270 are dubious. Pliny's description of Nero looking at the gladiatorial combats in an emerald means at best only a lorgnette, or most probably a reflecting mirror. Roger Bacon seems to have known of magnifying lenses (1276), which soon became common enough, but the probable inventor of spectacles as such was a Florentine worthy on whose tombstone in the church of Santa Croce is the inscription: "Here lies Salvino d'Armato degli Armati of Florence, the inventor of spectacles. May God forgive his sins. [He died] Anno Domini 1317."

Early in the fourteenth century, spectacles were mentioned in the writings of Bernard de Gordon, Arnold of Villanova and Guy de Chauliac, and they were afterward figured in the pictures and public documents of the period, such as Jan van Eyck's Madonna at Bruges, Martin Schöngauer's engraving of the Death of Mary, the decorations of the altar of St. Jacob's Church at Rothenburg an der Tauber or the drawings in a Ratisbon manuscript of 1600, now in the Germanic Museum at Nuremberg. All these indicate huge circular lenses mounted in rings of black horn or leather, united by a short leather band and fastened by another band passing around the head, the lorgnette and pince-nez patterns with metal mounts appearing later.

Prof. R. Greeff of Berlin, after a long search in different museums and collections has at length found the earliest known specimens of the old leather-mounted type of the sixteenth century. These are now to be seen in the Pirkheimer room in the Wartburg (near Eisenach, Thuringia), and were discovered behind the wooden wainscoting of Willibald Pirkheimer's chamber at Nuremberg in 1867. Pirkheimer's spectacles consist of eight pairs, the lenses mostly sprung or broken, and clouded through some changes in the glass. The eye-glasses of this period were called "nose-riders" because they straddled the nose and had to be supported by the hand from the side or above when used for reading. They were very expensive, says *The Journal of the American Medical Association*, costing from $45 to $75 a pair, and must have been a costly layout for even a wealthy Nuremberg patrician of the sixteenth century.

A NEW COLOR WAVE-LENGTH METER.

By Reinhard A. Wetzel,
College of the City of New York.

For many purposes in the laboratory and in educational physics the first three figures of spectral wave lengths are sufficient. An

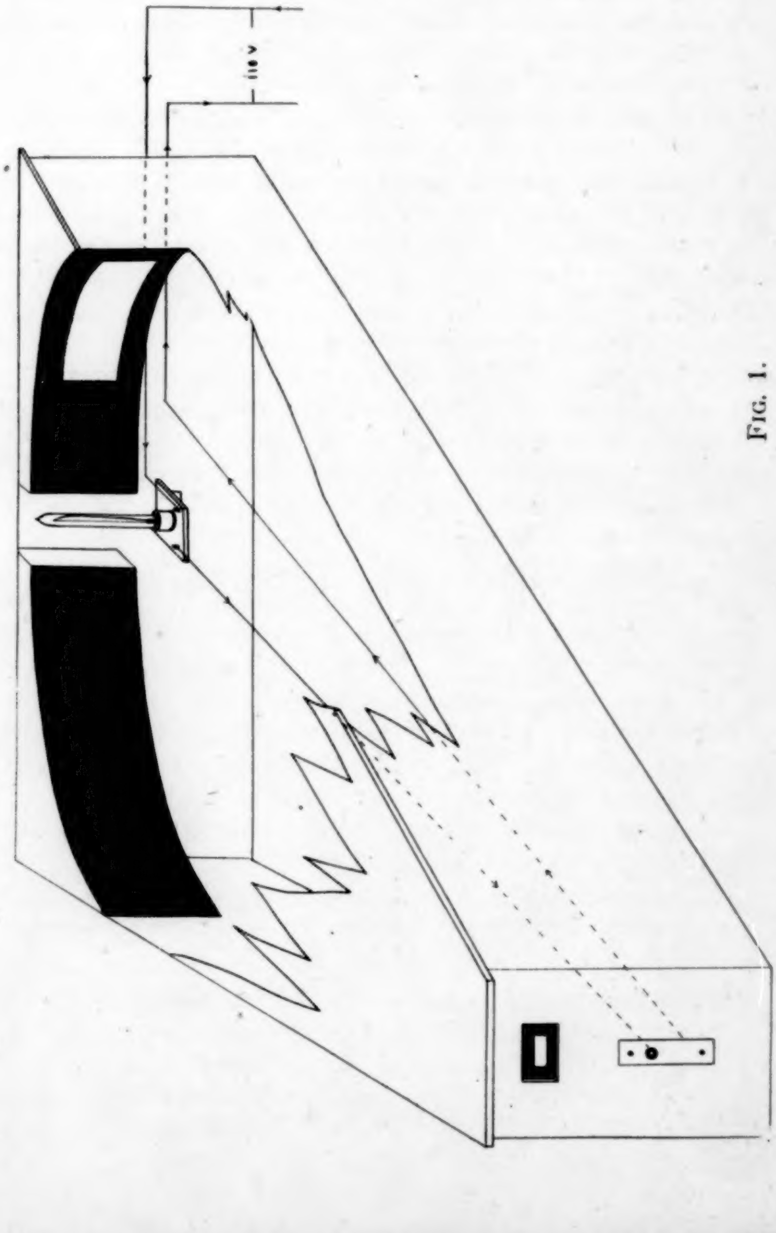


Fig. 1.

insufficient number of spectrometers for class use led to the development of the apparatus described.

The instrument evolved has been called a color wave-length meter from the fact that when we press an electrical button on a box thirty inches each way and fifteen inches deep, and look into the box through a small window, the eye sees two beautiful spectra superimposed upon a scale which tells the wave-lengths of the different colors, either in millionths of a centimeter or millionths of an inch.

The mechanism of the meter is very simple. For the window of the box we use a Rowland diffraction grating replica, 14,430 lines to the inch, which can be bought for as low a price as one dollar. At the opposite end and inside of the dark box is a straight filament carbon incandescent lamp costing seventy-five cents. Pushing the button on the box closes the electrical circuit through the lamp. A sixteen candle power line of light falls upon the grating which breaks a portion of the beam of white light up into spectra of the first and second order. These spectra may be received upon a cardboard screen or, better still, the retina of the eye. The divergent line of light falls upon the grating uncollimated. For all the eye can tell the spectral rays seem to come from a point within the dark enclosure.

To measure the wave-length of the colored lines in the spectra of gases the cover of the box is removed, the lamp unscrewed from its socket and a spectrum or Pluecker tube is suspended from a clamp stand in the position of the lamp. If the room is partially darkened the wave-length of the seven lines in helium, or the score in the spectrum of neon, may be read off as quickly as one would read them from a printed page. As the scale divisions are all of the same size, the spectrum tube need not be accurately centered if we take the average of readings left and right. The spectrum tube may be excited by the secondary circuit from an induction coil, or if an alternating current is available a small transformer is ideal for the purpose.

The accuracy of the meter may be observed by comparing a few readings. The eye which made the observations given was far-sighted; a normal eye could doubtless do better.

WAVE-LENGTH OF THE COLORS IN THE HELIUM SPECTRUM IN MILLIONTHS OF A CENTIMETER.

	Observed in Meter	Collie's Value	A Wilson Spectrometer
Weak red	70.8	70.65	Not visible
Bright red	66.8	66.77	67.08

	Observed in Meter	Collie's Value	A Wilson Spectrometer
Blinding yellow	58.4	58.76	58.95
Strong green	49.9	50.16	50.29
Medium green	49.1	49.22	49.40
Strong blue	47.2	47.13	47.29
Broad blue	44.4	44.72	44.77

If we compare the meter and spectrometer values with the accepted values (Collie: Pro. Roy. Soc. 71: 25—1902) it will be observed that the directly read meter wave-lengths are as correct as those obtained with a forty dollar spectrometer.

The following wave-lengths were read off in a neon spectrum. Opposite these are given the corresponding wave-lengths taken from H. Erdman's "Anorganische Chemie," page 214 (1902):

	Wave meter	Erdman
Red	70.6×10^{-6} cm.	70.3×10^{-6} cm.
	69.7	69.3
	67.0	67.2
	66.9	66.8
	65.9	65.9
	64.9	65.0
	63.8	64.1
	63.1	63.4
	62.5	62.6
Orange	61.2	61.4
	60.4	60.4
Yellow	59.2	59.5
	58.3	58.5
Green	53.8	54.0
	53.2	53.2
	52.0	52.0
	51.2	51.4

For educational purposes these figures are amply sufficient. More figures lead to confusion. The millionth of a centimeter is easy to remember and less confusing than the Greek mu, micron, tenth meter and Angstrom units. Experience with this direct-reading-scale also leads to an enlarged concept of micrometric dimensions.

The meter has another advantage over the spectrometer beside that of cost. There is no telescope to adjust for infinite distance, no collimator to adjust for parallel rays, and no slit to slide away from view of the source. There are no angles to be read off with a microscope and vernier; no tables of circular functions are needed—the only mistake possible is in observation, which can be corrected by repetition.

For the benefit of anyone who may desire to build his own apparatus, the instrument and method of calibrating the wave-length scale will be described more in detail. In a darkened laboratory where table partitions are provided, each pupil in a class might have his own meter in skeleton form.

THE DIFFRACTION GRATING

above described is quite satisfactory, but if one could be obtained with a greater number of lines per inch than the Rowland replica with 14,430, one that would separate the D line in sodium vapor and bring out the Fraunhofer lines in the beam of sunlight, it would be preferable. With the dollar grating, even on a spectroscope it has not been possible to show the dark lines in sunlight with a distinctness meriting a student's appreciation.

THE CONTINUOUS SPECTRUM

is best produced by the straight-filament carbon incandescent electric lamp. As this type of lamp is exceedingly useful for all projection work and galvanometer mirror deflection, it deserves to be better known. As straight-filament lamps can be had for any voltage and illuminate equally well with direct or alternating currents, there are few schools where they cannot be used. The lamp has but one socket, which fits readily into any of the standard incandescent lamp sockets. They cost little more than the ordinary lamps and their energy emission in comparison with the old sheet iron slit in front of a gas flame, is enormous. For spectral work the carbon filament is to be preferred to the more expensive drawn tungsten.

There are on the market at present two different types of lamps, each of which has its respective merits. The German type was obtained through the apparatus manufacturer, Max Kohl, Chemnitz. The current from socket to tip traverses a straight carbon filament and returns to the other pole of the socket by means of a heavy metal conductor. The cold conductor has the shape of a bow under tension, which takes up the elongation when the carbon filament becomes incandescent. The American O'Brien lamp, made by the Straight-Filament Lamp Co., New York City, is similar in principle, the current returning from the tip of the lamp through a fine wire, which is surrounded by a capillary glass tube. At full voltage the enclosed wire conductor becomes dull red, but its intensity is not sufficient to mar the spectrum. This hot wire may be avoided, however, for at half the marked voltage the lamp still gives sufficient light for a spectrum somewhat diminished in brilliancy. In the German type the return wire is of such a size that it does not become heated and is painted black to prevent reflection. The six-inch "candelabrum" of the Straight-Filament Lamp Co. may be a little more fragile than the German lamp, but its sixteen candle power

emission produces a superior spectrum. Figure 2 shows the lamps in longitudinal section.

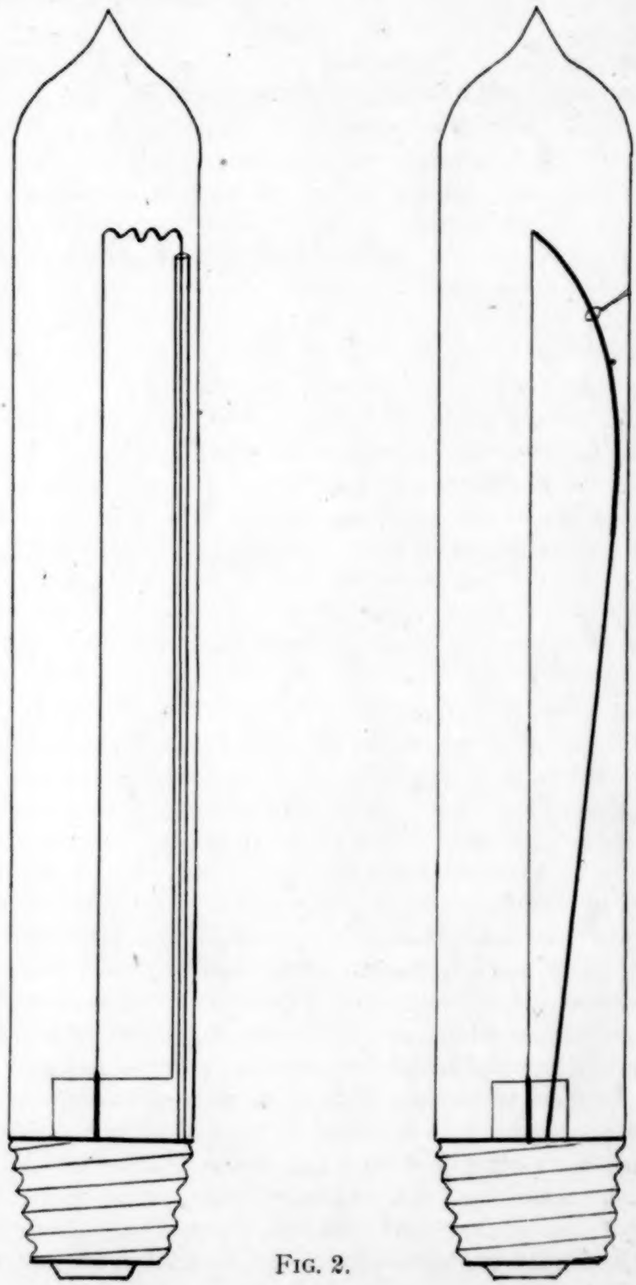


FIG. 2.

On looking at the spectral lines of a gas which are obtained from a transmission grating held close to the eye, considerable parallax will be observed if the virtual spectrum falls upon a distant scale. That is, if the eye moves over the grating from left to right the lines of the spectrum will move with respect to the scale, either in the same or in the opposite direction. By moving the scale forward or backward, a place of no parallax will be found. If the light comes from a helium tube, the non-parallax position may be found for each of the seven bright lines.

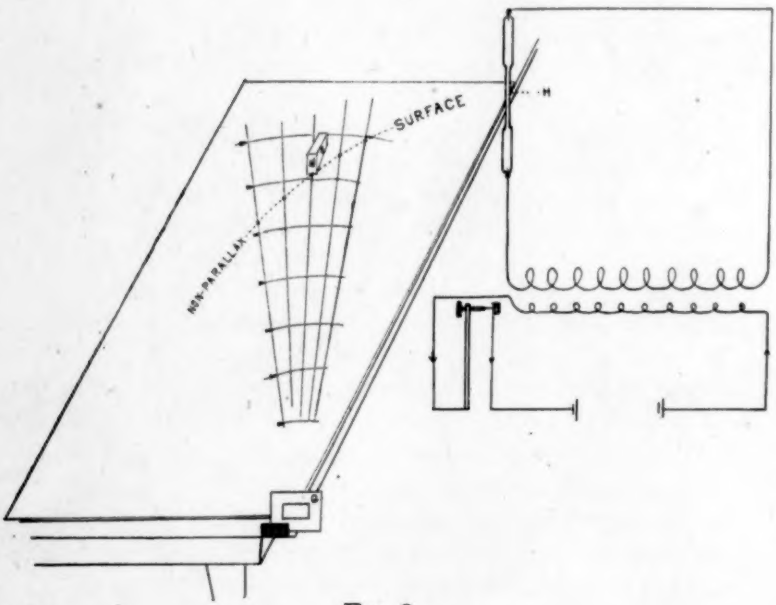


FIG. 3.

Figure 3 shows a good way to record our observations on a plain sheet of paper fastened to the top of a table. The helium tube is placed near the edge of the table; as a matter of uniformity, exactly 100 cm. from the table-corner. The grating is fastened in its upright position to the corner of the table by means of a clamp or tacks, so that the incident beam is normal to the center of the grating.

From the center of the grating draw radii along which the various spectral lines are visible. For convenience of record, concentric circles are drawn, say a centimeter apart. In figure 3, S is a block for holding upright a small black cardboard screen

on which a few perpendicular parallel lines are drawn in white. Larrabee's white ink is very useful for optometric rulings. G represents the grating.

The method of work is as follows: Place the screen on the virtual path of the red ray. Move the eye over the grating from left to right. Did the image move with respect to the rulings on the screen? Which way and how much? Record the fact and the screen's distance at which this happens. Try other positions along the path of the red ray. It will soon be observed that there is a point where the direction of image-motion reverses with respect to the rulings on the screen. This is the non-parallax point for the red ray and is marked with a cross upon the graphing paper.

In a similar manner the non-parallax points may be found for the other six spectral lines. Through all the non-parallax points a smooth curve is drawn. The spectrum on the opposite side of the incident beam may be investigated in a similar manner. The non-parallax surface is symmetrical with that on the other side. The non-parallax surface is then sawed out of a heavy wooden board, which serves to hold the scale which we will now proceed to calibrate.

A narrow strip of unruled paper is pinned to the non-parallax surface after it has been firmly fastened in its position immediately behind the spectrum tube. When the eye is at the grating a pencil is held so that its point corresponds to the position of a line in the spectrum. In a semi-dark room both pencil and line may easily be seen in coincidence. Thus it is easy to record the exact location of each line. For accuracy a number of such record strips are made and carefully measured up. Distances on the non-parallax surface measured from the lamp are most convenient. Since the accepted wave-length of the colors in the helium spectrum are known (see above) the ratio between the known wave-length and its actual distance from the lamp on the non-parallax surface can be determined for each observation made.

To obtain the location of 10, 20, 30, 40, 50, 60, 70, 80 millionths of a centimeter on the non-parallax surface, we divide each of these numbers by the above average ratio. It is then easy to subdivide the divisions into ten equal parts, and our work is done. Black cardboard with rulings in white ink makes a good scale. The scale in millionths of an inch may be derived in the same way. A scale simply measuring degrees of diffraction may be

made, by the use of which the student can calculate the wave-length L, from the expression

$$L=E \sin x$$

where E is the distance between the centers of two consecutive rulings on the grating, and x the angle between the diffracted ray and the incident light coming from the source.

EQUIPMENT FOR A GROUP OF STUDENTS IN A LABORATORY.

The experiment is quite successful in a semi-dark room with simply the skeleton scale, lamp and grating. With the scale calibrated for 100 cm. distance, a group of students experiences no more difficulty in performing this experiment than one in photometry.

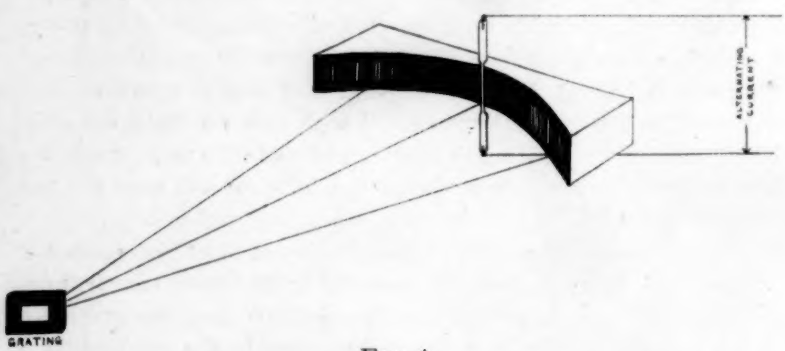


FIG. 4.

Figure 4 illustrates the skeleton set-up.

Sometimes the scale divisions between 0 and 30 on the metric scale are bright enough to act as separate sources of light, the diffuse spectra of which interfere with the reading of the scale. To avoid this, these numbers may be omitted from the calibrated scale, leaving a perfectly black background behind the lamp.

NEW YORK STATE SCIENCE TEACHERS' ASSOCIATION.

This association will hold its 1912 meeting at Syracuse, December 26, 27 and 28. A most excellent program has been prepared. It will be helpful to every one who attends. All science teachers abreast of the times should be there. Go prepared to take part in the discussions.

THE STATISTICAL INVESTIGATION OF SCHOOL GRADES.

By W. L. Eikenberry.
University High School, University of Chicago.

In the study of error and chance it has been shown that errors are amenable to a mathematical law. For instance, if one were to toss a penny at a crack a great many times it might be confidently predicted that he would not hit the crack every time. It might also be predicted that the coin would alight sometimes on one side of the crack and sometimes on the other. Indeed both theory and experiment agree in the conclusion that if there were no disturbing factors, the distribution of the casts would be equal on the two sides of the crack and the number of occurrences at any particular distance would bear an inverse relation to the distance. That is, the coin would fall most frequently near the crack and proportionally less frequently at greater distances. If the frequency of the fall of the coin at different distances to right and left be ascertained and these frequencies be represented graphically by plotting the curve it is found that the resultant curve is a symmetrical one. It reaches its highest point on the axis which represents the frequency upon the crack, and it descends equally to right and left. This curve is found to correspond very closely to the curve derived from the binomial equation (see dotted line, Fig. 3). Also, as the number of casts becomes greater the correspondence becomes closer and we are compelled to believe that for a very large number of casts the correspondence would be practically a coincidence. This curve is called the curve of the probability of error, or sometimes in biological statistics, the normal curve.

Study of the variation of various single characters in plants and animals has shown that these variations are in general distributed about a central mode in a fashion quite in harmony with the distribution of errors. There are of course many deviations from this standard distribution and each such deviation becomes a problem in itself. Both types of distribution are conveniently illustrated in the curves showing the variation of number of grains per head in two types of wheat, published in School Science and Mathematics, Vol. II, No. 1, page 35, 1911. Curve "A" corresponds very closely with the binomial curve and is such a figure as one would ordinarily expect to find; curve "B" is very aberrant and suggests that it may be in fact composed of several regular but overlapping curves. The symmetrical curve,

"A", shows that in this variety of wheat there are a few small heads, a great many of medium size, and a few very large ones. That is, most of the individuals are mediocre and a few are exceptional in either direction. This law holds generally throughout the plant and animal kingdoms. With respect to any particular character it is found that most of the individuals of a species fall within mediocrity,—the central part of the curve—with a smaller number of exceptional individuals on either side of this central mode, and that the curve of distribution approximates the curve of probability of error. The exceptions are amenable to explanations which are consistent with the law.

If it be admitted that the probability curve is the normal variation curve in plants and animals, it follows as a corollary that variation in the physical characteristics of man should have a similar distribution. Data bearing upon this question have been collected and studied by many. To instance only one, Karl Pearson made a study of the stature of English women founded upon 1025 individuals. The resultant curve approximates the theoretical curve very closely indeed. (Fig. 1.) The correspondence is of as high degree as theoretical considerations would lead us to expect. Other studies of a similar character serve only to strengthen the conviction that in this respect man is amenable to the general biological law.

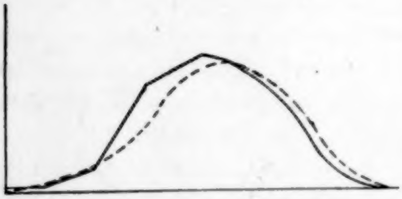


FIG. 1.

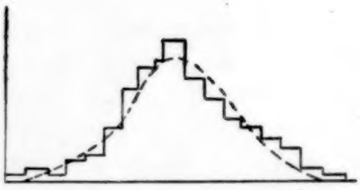


FIG. 2.

The notion that the law of biological variation may be properly applied to the study of mental and moral characters is almost inevitable. In a study of this sort there is of course a primary difficulty resulting from the imperfection of our methods of measuring such characters. Attempts at such measurement have however been made with, it is believed, a fair degree of accuracy, and considerable of the newer work in psychology is directed towards the accumulation of such measurements and the perfection of the technique needed in making them. Thorndike has

reported the results of a test of the efficiency of certain pupils in rapidity and accuracy of perception. The resultant curve again closely approximates the probability curve though as might have been expected not so closely as in the case of curves representing the more easily measurable physical characters. (Fig. 2.) The same thing is illustrated with a fair degree of correspondence in a study of memory for related words. Other similar studies have been made in sufficient numbers to lend great probability to the notion that mental characters do not differ in respect to the laws of their variations from physical characters. It is commonly assumed in all such studies that the normal distribution is that which corresponds to the probability curve.

School grades constitute a great body of data bearing upon the matter of individual mental abilities—possibly the greatest body of such data now available. Each grade represents an effort to measure the results of mental effort. In two particulars at least these grades are not ideal material. In the first place they do not pretend to measure ability, primarily, but achievement and it is quite possible that in a certain fraction of the cases the results achieved are not in close correspondence with the intrinsic ability of the individual; they may be due to unusual effort or to unusual carelessness. Doubtless both of these cases exist in any body of school records in considerable numbers but it is also probable that they occur in nearly equal numbers on both sides of the equation and practically counterbalance each other. At any rate, any investigation of school matters is quite as likely to be concerned with results as with abilities. The experience of investigators is that this factor does not in practice seriously modify the results.

A second objection lies in the possible unequal or wholly incommensurate standards of different teachers. If data are collected sufficiently widely to include a great many teachers this divergence also will disappear. Very frequently it is not sought to eliminate this deviation due to the individuality of the teacher; it is desired on the contrary to use the facts for the purpose of evaluating the teacher. The best evidence that the normal distribution of school grades is in accordance with the probability curve is the fact that whenever large numbers of typical records are studied they are found to be distributed in accordance with the theoretical considerations. As illustrations of this we may cite the study of Dearborn on the averages of 472 high school pupils (Fig. 3) and Johnson's study of the grades given dur-

ing two years in the University High School. The latter study includes all grades which are matters of record during the given period making a total of 13,726 grades. A later study, including over 25,000 grades is shown in Fig. 4. The close correspond-

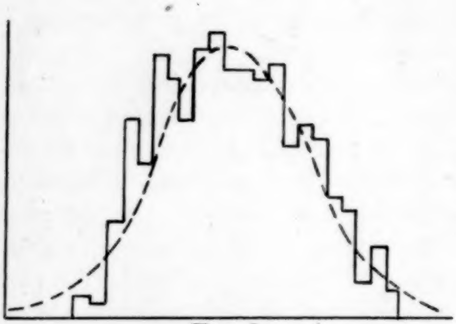


FIG. 3.

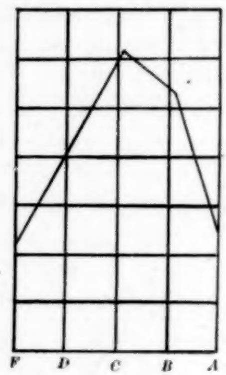


FIG. 4.

ence to the mathematical curve in all three cases is notable. The general correspondence in studies of this character is so close that the relation is now considered to be rather firmly established, and any serious deviations from this normal distribution are legitimate subjects of investigation.

Once it is granted that the law of variation holds in the main as to school grades, the way is open to investigate the work of departments, schools and individual teachers. A new and valuable tool has been gained. It is now possible to make quantitive and detailed comparisons between departments and between a department and the standard. Deviations from the theoretical distribution of marks may be entirely justifiable; indeed the circumstances may be such as to demand them as when the limitations of the course of study make the pupils of a class or department a highly selected group, but always a deviation from the normal raises a pertinent question calling for answer. Possibly the greatest value of such statistical study does not lie in the information conveyed by the simple curve of distribution but rather in the secondary investigations which are demanded in order to account for irregularities which appear. At the present stage in our knowledge of the exact condition of grading and indeed of most school affairs a new method of investigation, a new set of results, is likely to be of value precisely in proportion to the

stimulus which it gives to farther investigation and the definiteness with which it defines the starting point and direction of the needed investigation. Statistical examination of any set of grades does just this.

As a concrete example of such a study we may refer again to Johnson's "A Study of High School Grades" cited above. As examples of departmental studies there are presented here his figures showing conditions in four departments. (Fig. 5-8.)

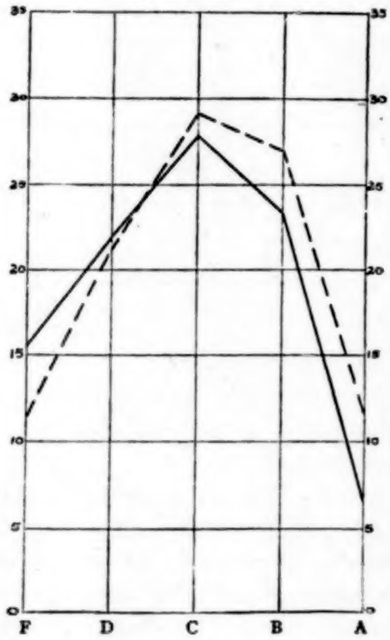


FIG. 5.

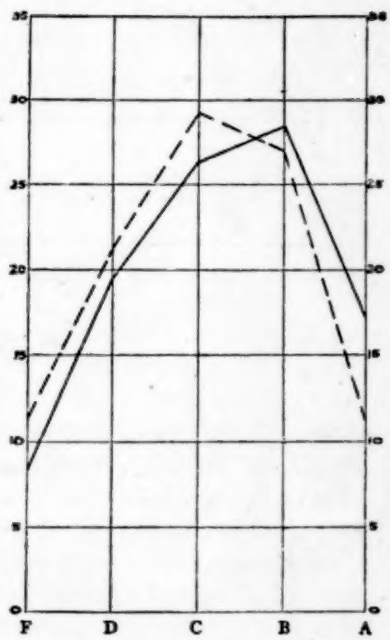


FIG. 6.

In each case the dotted line indicates the average of the school and the department curves may be compared directly with the same standard. All of these curves are constructed upon the basis of percentages and are therefore comparable. It will be noted that in fig. 5 there is an excess of D and F grades; in fig. 6 an excess of A and B; in fig. 7 a large deficiency of B grades and an excess of D and F; while in fig 8 the most marked peculiarity is a very large proportion of B grades. In all of these cases the form of the curve relates it to the theoretical curve but it is shifted bodily to right or left. As to the interpretation

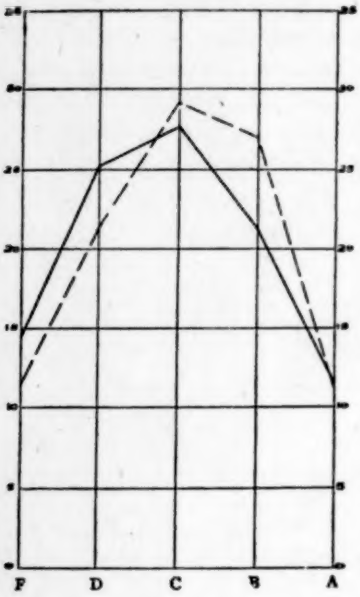


FIG. 7.

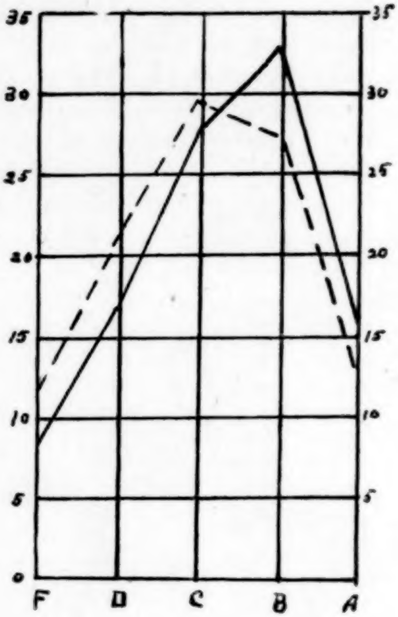


FIG. 8.

of this displacement, at least three explanations present themselves. If the curve is displaced toward the "F" end of the scale as in the case of fig. 7, it may be that the standard of the work expected of the pupils is higher than the age and ability of the pupils would justify; it may be that the material of the course is properly selected but that the instruction is relatively inefficient; or it may be that the standard of marking is too high. When the curve is displaced to the right as in fig. 8 the converse of these explanations would apply.

The comparison of individual teachers becomes a great deal more personal than the investigation of departments though even the latter is quite capable of arousing considerable individual interest. Two graphs from Johnson's paper may serve to illustrate individual peculiarities. One (Fig. 9) shows the divergence between the standards of two teachers within a single department when pupils and subject matter are as nearly uniform as possible and the other (Fig. 10) shows even greater divergence in a case where two departments are involved. One teacher, appearing in both figures, gives a maximum of high marks and a minimum of low ones while both of the other teachers group nearly half

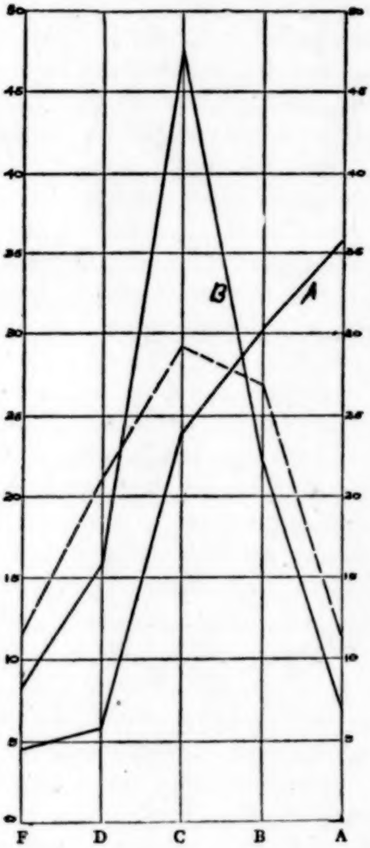


FIG. 9.

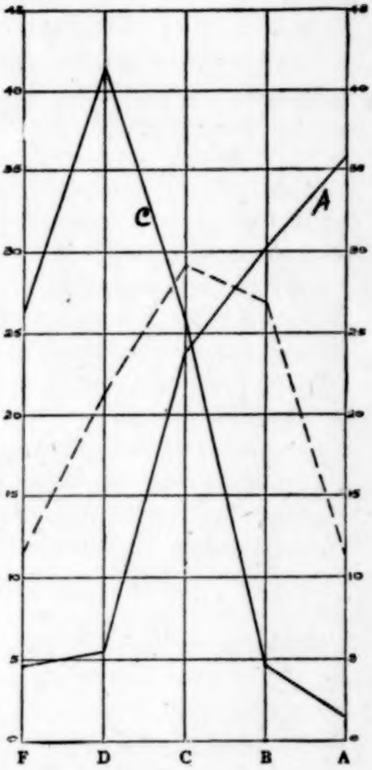


FIG. 10.

of the whole number in a single rather low grade. Evidently the subject matter cannot be so largely at fault for the matter of instruction is usually a department concern and there is not opportunity for such large variation, particularly in the case of two teachers in the same department. To take a particular case, let us consider the set of grades which find their greatest frequency in the D column (curve C). That this teacher's classroom is sought most largely by the lazy and incompetent is not probable and would doubtless be denied by the individual. The incompetents do not seek a teacher with a record for low grades. If then his pupils were of average ability it is not admissible to believe that only 2 per cent of them actually succeeded in accomplishing the full amount of work that might fairly be expect-

ed of them and therefore deserve A, nor is it easy to believe that more than 40 per cent of them succeeded in accomplishing only the minimum and 27 per cent failed entirely to measure up to expectations. To put it in another way, we may assume the grade "C" to express mediocrity, and in that case 80 per cent are alleged to be *below mediocrity*—rather a contradiction in terms. The only adequate explanation seems to involve either a correct standard of marking with inefficient teaching, or average teaching with an impossible standard. At any rate it can hardly be doubted that there is a large divergence in the standards of these several teachers and that this divergence works injustice to the pupils and to the teachers.

In certain institutions where this subject has been given a good deal of attention efforts are being made to so standardize the marking system that in the long run the distribution will be approximately the same for all teachers in the school. There seems to be a rather common agreement that the use of five grades the lowest of which indicates failure is convenient and scientifically acceptable. There is not yet such common agreement regarding the boundaries of the groups, particularly as to the number of pupils which we may expect to find the first and last groups.

Such statistical study is likely to prove of very distinct service to a new teacher entering a school for the new teacher is always at loss to know what difference there may be if any in the capacity of the new group compared with the old, and what difference in the school standard of marking. In the absence of any method of conveying this information from the administrative department, the first part of a new teacher's work becomes a sort of cut and try process. The statistical method will make it practicable for the school officers to give a new teacher some effective guidance in this rather difficult situation.

The writer may be pardoned for illustrating this point by reference to a study of the grades given by him in the University High School since other data are obviously not easily secured. Such study was particularly important to the writer because conditions in the University High School are very different in certain particulars from those obtaining in the public high school in which he had worked for some six years previously. In the latter school the author's courses were in the first year and were required of all pupils, the pupils came from widely divergent social conditions and were looking forward to many

different occupations with but few expecting to attend college. The University High School draws its pupils from a different and more homogeneous social group, the courses concerned are in part elective, and the majority of the pupils are expecting to enter college. The fact that the University High School is a tuition school is also a factor of considerable importance. Besides all this slightly different values are assigned to the letters used to indicate grades.

Recognizing these differences in the two schools but without positive data as to their effects, the author found considerable uncertainty in the matter of grading. Careful observation of the new conditions served to correct the more pronounced tendencies which were found in the grades given at first, but no amount of general observation would suffice to assure any very close approximation to the standard of the school. At the end of the second year an attempt was made to measure the degree of correspondence between the school standard and the individual standard. This study could not have been carried on successfully at an earlier time because there were not on record sufficient number of grades to make a normal distribution. This time was a favorable one for the study both because in the two years sufficient number of grades had accumulated to make a fairly typical distribution, and also because during these two years an intensive study of all the grades of the school for several years had been made by the Principal, and the distribution curve for the school and for each department was known.

The number of grades of record given by the writer during the two-year period studied is 295. Their distribution is shown graphically in Fig. 11, in which curve "C" represents the grades in question, while "B" represents the distribution in the science departments and "A" is the school average.. It is of course perfectly obvious that while the number failing is almost the same as in other sciences and not widely different from the school average, the same does not hold true regarding other grades. There is a great preponderance in A and B grades with the corresponding deficiency in C and D grades, though the divergence from the practice of the science departments is not so great as the departure from the practice of the whole school. It is needless to say that this comparison has occasioned a very considerable revision of standards and methods as such a study may be expected to do in most cases.

While the writer is frank to admit that in this case the graph

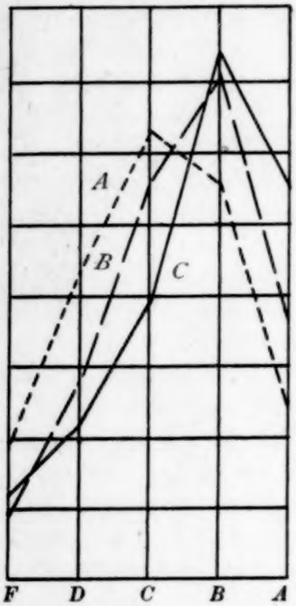


FIG. 11.

shown indicated a change in both standard of grading and material of instruction, it does not follow that this is always the case. There may be cases, particularly if the class be small, when anything else than a preponderance of high grades would be a great injustice. What it is intended to insist upon is that any great deviation from the normal should be the subject of careful consideration on the part of the teacher concerned, and that the teacher who gives an unusual number of high or low grades should make sure that he knows why he does so in each case. If it is certain that the reason lies in the characteristics of the pupil the correction must begin there, but if it lies in the subject matter or the standard of grading it is these that need correction. It can scarcely be doubted that a careful statistical examination by every teacher of his own results would tend strongly toward more uniformity of grading.

A QUANTITATIVE EXPERIMENT FOR GENERAL CHEMISTRY.

By Robert W. Curtis.
College of the City of New York.

An examination of the many laboratory manuals for elementary chemistry with a view to determine the ratio of the number of quantitative experiments to the whole number, would reveal a considerable variation—perhaps from 100 per cent to 0 per cent. Not long ago much stress was laid upon the quantitative experiments in the course— they were evidently held to possess a high pedagogical value. The teacher of small classes has especially felt this benefit, for the quantitative experiment gives good opportunity for quizzing and is notably serviceable for illustrating laws. In the hands of large classes, however, there is danger in the employment of a large proportion of quantitative work, and the experiments chosen must be capable of yielding good results in order that the work as a whole may be beneficial.

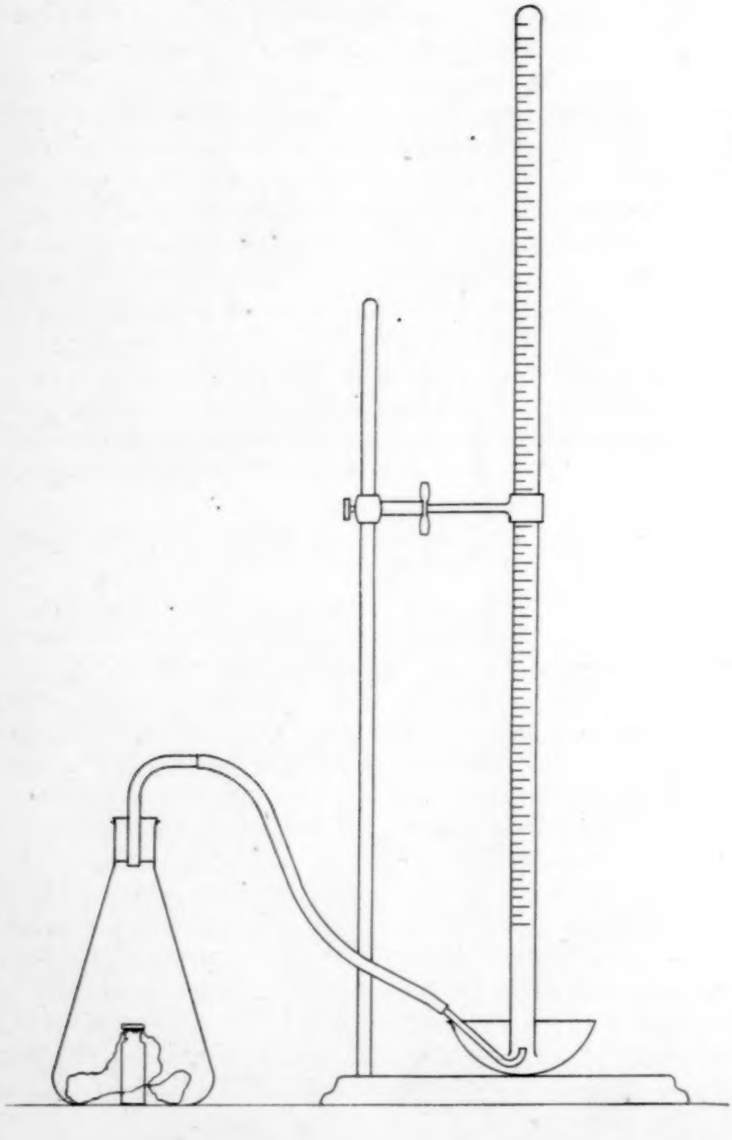
In adapting a quantitative experiment for elementary work, a critical point is the compromise that must be made between approved quantitative procedure on the one hand, and necessarily simplified operations suitable for the beginner, on the other. The danger here is that, in the effort to get good quantitative results and inculcate good habits of manipulation, so many details and devices are introduced as to obscure the main point of the experiment.

A quantitative experiment involving the measuring of a gas has several commendable features, among which are, opportunity to apply concretely the gas laws, passing from volume to weight, the use of an already familiar method of gas generation and the attractiveness of such an experiment. It is not surprising then, that measuring the hydrogen evolved in the reaction of magnesium with dilute acid is a favorite experiment with many teachers and is to be found with many variations of detail in most laboratory manuals.

There is presented herewith a form of the experiment adopted for use in large classes, in which is employed a simple apparatus making use of the principle of displacement of the Victor Meyer apparatus, together with some figures obtained in working out conditions, and some from the results of the students themselves which may afford criteria for judging requirements of accuracy in an experiment of this kind. The arrangement has the ad-

vantage that the magnesium is enclosed, thus preventing loss and is under direct observation so that one can be sure that it is completely dissolved before reading the volume of the hydrogen.

Determination of the Weight Relation Between Hydrogen and Magnesium in the Reaction Between Dilute Sulphuric Acid and Magnesium.



Obtain a 100 cc. eudiometer, Erlenmeyer flask, delivery tube, homeopathic bottle, piece of ¼-in. flexible rubber tubing, a piece of string on "Temporary Loan" at the stockroom.

Examine the eudiometer to see how it is graduated—the value of the smallest divisions. Observe that when it is set up the numbers increase downwards.

Set up eudiometer as shown in model (see cut p. —).

Take a piece of magnesium ribbon and measure carefully (1) its length. Calculate (2) its weight, using weight of unit length as given on blackboard. Record all measurements clearly and systematically in notebook.

Rinse the Erlenmeyer flask, the homeopathic bottle and your graduated cylinder thoroughly so as to be sure they are free from any acid.

Pour about 50 cc. of water into the Erlenmeyer flask.

Fold up the magnesium ribbon and drop it into the water in the Erlenmeyer flask.

Tie each end of the string around the neck of the homeopathic bottle. (See blackboard.) Fill the bottle to the neck with dilute sulphuric acid. See that the delivery tube is away from the mouth of the eudiometer.

Lower the bottle containing the acid, carefully, into the Erlenmeyer flask and let it stand on the bottom, dropping the string in the flask.

Place the stopper in the mouth of the flask, and then the delivery tube under the mouth of the eudiometer. Touch the flask when necessary only near the stopper. (Why?)

Incline the flask cautiously, allowing the bottle to fall over and thus mix the acid with the water.

When all action has ceased and the magnesium has been completely dissolved, carry the eudiometer, holding it by the clamp and keeping the end under water in a crucible, to a large glass cylinder containing water in which it may be held so that the surfaces of the water inside and outside the eudiometer are on the same level. Read the volume (3).

Record also (4) the pressure of the atmosphere as indicated by the barometer; (5) the temperature of the gas in the eudiometer (this may be considered the temperature of the room) and (6) the pressure exerted by the water vapor in the gas in the eudiometer, sometimes called "aqueous tension." (This may be found from a table, the value depending upon the temperature.)

Now from these data calculate Result A: the weight of hydro-

gen obtained. (The weight of a liter of Hydrogen at 0° and 760 mm. may be taken as 0.0899 g.)

Calculate Result B: The weight of hydrogen obtainable by the reaction with *one* gramme of magnesium, using for this calculation the weight of magnesium taken and Result A.

In the heading, all reference to "equivalent," molecular weight, etc., is omitted, the idea being to concentrate the attention upon the one fact of a quantitative relation, it being left for the class room to develop theoretical conceptions which may be done advantageously by introducing some of the students' own results for data.

An approximate weight of the magnesium is taken, obtained by multiplying the length by the weight of a unit length, avoiding the use of balances. This is done because it was thought that instruction in the use of the balance at this time tends to make the work of the experiment too extensive, requiring so much time as to prevent completion in one period of two hours. To ascertain the effect of this expedient upon results, a number of experiments were made under the direct supervision of the instructor,[1] using different samples of the same magnesium ribbon as that used by the class, as shown.

TABLE I.—GAS FROM UNIT LENGTH AND UNIT WEIGHT OF RIBBON.

Ribbon	No. Expts.	Av. Vol. 0°760mm fm. 1cm	Av. % Var. fm. mean	Av. Vol. 0°760mm fm. 1gm	Av. % Var. fm. mean
A	10	5.32cc	4.4	1016cc	4.7
B	11	5.15	5.6	917	6.0
C	11	5.05	6.1	957	5.8
ABC	32	5.17	5.73	962	6.43

The results indicate that certainly nothing is lost in accuracy by this procedure.

The following table gives an indication of the uniformity of the ribbon—showing considerable variation.

[1]Students Aronson and Abrams kindly gave assistance in this work.

TABLE II.—WEIGHT OF UNIT LENGTH MAGNESIUM RIBBON.

Ribbon	No. Samples	Mean Wt. 1^{cm}	Av. Var. fm. mean	Per cent. Var.
A	14	0.00523^{gm}	0.00003^{gm}	0.57
B	12	0.00557	0.00004	0.72

Among the numerous "sources of error" in this laboratory exercise it was found that the heat of dilution of the acid was a considerable one. When using 75cc. of water and 5cc. of concentrated acid there was an average increase of 6°.4 (23 experiments). This corresponds to 3.2cc. in the volume of an average experiment. By substituting 50cc. of water and a bottle full (about 13cc.) of dilute acid (1:5) this effect is reduced to 1°.8, corresponding to an error of 0.9cc. in volume.

Again, to keep the *modus* of the exercise as simple as possible, the barometer reading is used without correcting for expansion of mercury column. The error from this source is calculated to be 0.3cc. of gas in a typical experiment.

In order to choose the fairest place to obtain the temperature to be taken for the experiment, when carrying out the thirty-two experiments of Table I. thermometers (previously compared) were placed (a) in the generating flask, (b) by the eudiometer, (c) in the room, (d) in the water of the tall cylinder. A consideration of the readings from these, which were in substantial agreement, (except (a) as already noted) led to the conclusion that the room temperature is by all means the most satisfactory.

In order to analyze the students' results, these with their data were transferred to cards to facilitate checking and classifying. First, all reductions of gas volumes to standard conditions were checked as to correctness with the following showing:

TABLE III.—ERRORS IN REDUCTIONS.

	Group A.		Group B.		Group C.		Group D.		All	
Error	No.	%	No.	%	No.	%	No.	%	No.	%
0	30	46	13	38	9	33	20	42	72	41
0.1	8	12	2	6	3	11	4	8	17	10
0 and 0.1	38	58	15	44	12	44	24	50	89	51

The correctly reduced volume was then divided by the length of the ribbon used to obtain a showing of the constancy of results.

TABLE IV.—GAS AT 0° 760MM. DERIVED FROM 1CM. OF RIBBON.

Group	No. Students	Average
A	72	4.20^{cc}
B	41	4.54
C	23	5.00
D	65	5.51

By arranging all these values in order, it was seen that after rejecting about one-tenth of the results at each end of the series, manifestly caused by accident or exceedingly poor work, a fair average could be obtained from which constant value, new volumes were computed. The difference between this volume, based on the length of the students' ribbon and the constant volume per unit length ribbon, calculated for the conditions of temperature, pressure, etc., according to the student's data, and the volume actually obtained by the student, is regarded as the "error" and is exhibited in the following:

TABLE V.—ERRORS IN VOLUME OBTAINED.

Gas at Temperature, Pressure, etc., of Experiment.

Group	No. Students	Av. Error	Av. Vol. Gas Obtained
A	63	3.0cc	62cc
B	36	3.4	70
C	14	4.0	71
D	61	3.1	65

These errors were arranged by lengths of ribbon to see if it would appear that any special length gave better results. The average errors seem to be fairly constant indicating no advantage in using any particular length.

TABLE VI.—ERRORS BY LENGTH OF RIBBON.

Length	No.	Average Error
5-6 cm	2	5.5cc
6-7	4	2.7
7-8	3	3.8
8-9	1	1.0
9-10	3	2.9
10-11	42	2.9
11-12	23	2.9
12-13	20	3.7
13-14	29	2.9
14-15	39	3.6
15-16	9	3.7
16-	1	3.0

From observation of the students at work, large sources of error may be stated as accidental admission of air when transferring the eudiometer or in disregard of directions when inserting stopper in the flask, increase of volume from heat of hand, and mistaken reading of eudiometer.

Much interest was manifested in their work and it is believed that the *modus operandi* of the experiment is such as to appeal to the student's natural ingenuity.

MATHEMATICAL INSTRUCTION AND THE PROFESSORS OF MATHEMATICS IN THE FRENCH LYCEES FOR BOYS.[1]

By R. C. Archibald,

Professor of Mathematics at Brown University.

The general scheme of the French educational system and the position of the lycée in this system are topics, the consideration of which the title of my paper, strictly speaking, excludes. And yet, to give appropriate setting to the main themes and bases of comparison with our own schools, brief reference to these topics seems not wholly uncalled for in this connection.

For educational purposes France is divided geographically into *arrondissements*. The assemblage of government schools (primary, secondary and superior) in each arrondissement forms an *académie* over which a *recteur* presides. There are thus the 16 académies of Aix-Marseilles, Besancon, Bordeaux, Caen, Chambéry, Clermont, Dijon, Grenoble, Lille, Lyons, Montpellier, Nancy, Paris, Poitiers, Rennes, Toulouse, as well as a seventeenth at Algiers. With the exception of Chambéry these names correspond to the seats of the French universities.

The assemblage of académies forms the *Université de France,* at the head of which is the Minister of Public Instruction, who is *ex officio* the "Recteur de l'Académie de Paris et Grand Maitre de l'Université de Paris." For the Académie de Paris there is a vice-recteur, whose duties are the same as those of the recteurs of other académies. Although nominally lower in rank than the heads of académies in the provinces, he is in reality, the most powerful official in the educational system. The position of the Minister of Public Instruction being so insecure by reason of changing governments, continuity of scheme is assured by three lieutenants who have charge respectively of the primary, secondary, and superior education. They in turn have an army of inspectors who report on the work and capabilities of the recteurs and their académies as far as primary and secondary instruction is concerned.

This suffices at present to indicate the remarkably centralized and unique character of the French educational system. It is theoretically possible for the most radical changes in any part of

[1]Abridgment of a paper presented at the mid-winter meeting of the Association of Mathematical Teachers in New England, held at Brown University, Providence, R. I., February 3, 1912.

public instruction to be immediately brought about by a stroke of the pen on the part of the Minister of Public Instruction.

The present system of secondary education in France dates from the great reform of 1902 (important modifications were introducted in 1905 and 1909) and is carried on for the most part in *Lycées* and *Collèges communaux* which are to be found in nearly all cities. Because of their pre-eminence we shall consider the former only, which are under control of the state. Here the boys, who come from families in comfortable circumstances, may enter as *élèves* at the age of five or six years and be led along in their studies till they receive the *Baccalauréat* at the age of 16 or 17.[2] Many lycées have still more advanced courses to prepare for entrance into such schools as the École Normale Supérieure, École Polytechnique, École Centrale, École Navale, École de Saint Cyr, etc.

Instruction in fully equipped lycées may be divided into four sections:—I, *Primary;* II, *Premier Cycle;* III, *Second Cycle;* IV, *Classes de Mathématiques Spéciales.*

I.—*Primary.*[3] The classes in this section are named as follows:—

		Age from
Classes enfantines	Onzième	5
Classes préparatoires	Dixième	6
	Neuvième	7
Classes élémentaires	Huitième	8
	Septième	9

From the Dixième to the Septième 20 hours are devoted to class recitation each week. In the *Classes préparatories* 3 hours a week are taken up with *Calcul,* that is, principles of numeration, elementary operations with integers, notions concerning the metric system; intuitive geometry; simple exercises to enable the pupil to draw the more elementary regular figures (square,

[2]The pupils at the lycées are of four kinds: 1st.—*Externes,* those who come to the lycées for classes but board and lodge outside: 2nd.—*Internes* or *pensionnaires,* élèves who live entirely in the establishment; 3rd.—*Demi-pensionnaires* who usually reside at a distance but take their mid-day meal at the lycée; 4th.—*Exte'nes surveillés,* that is externes who work out their lessons under the eye of the *préparatéur* in the *salle d'étude* of the lycée. The expenses of the pupil vary greatly with the class and the lycée in which he happens to be. The range of cost (in francs per year) (1) for some of the principal cities (Bordeaux, Lyons, Marseilles, Toulouse) of the provinces and (2) for the better lycées of Paris is as follows: *Externes* (1) 70-450, (2) 90-700; *externes surveillés* (1) 110-540, (2) 130-790; *demi-pensionnaires* (1) 370-850, (2) 500-1200; *pensionnaires* (1) 700-1200, (2) 900-1700. The lower price in each case is for the classe enfantine, the higher for the special classes open to *bache'iers.* Primary education (outside of the lycées and superior education) in France, is free.

[3]*Free* primary instruction is given in *Écoles Primaries Élémentaires* for pupils from 6 or 7 to 13 years of age. The course is divided as follows: Cours élémentaire (2 years), cours moyen (2 years), cours supérieur (2 years). On completion of the cours moyen the pupil receives a *certificaté d'études primaries élémentaires.* This certificate or its equivalent is required of every child in France. Many children require considerably more than four years to get the *certificat.*

rectangle, triangle, circle) and different sorts of angles. In the *Classes élémentaires,* 4 hours a week are assigned to revision of the preceding programme; decimal numbers; rules of three; intuitive geometry by the aid of models. One hour a week is given up to drawing.

II.—*Premier Cycle (sixième-troisième).* This cycle of four years constitutes an advanced course for students who have finished their primary studies, and is the first part of secondary education proper. It offers a choice between two lines of study, the one characterised by instruction in Latin with or without Greek, the other in which no dead language is taught. The former is selected by the parent who wishes to prepare his boy for the department of letters in the École Normale Supérieure or for the career of classical professor, lawyer or doctor. The latter is likely to be chosen for the boy who is particularly interested in science or who has a commercial career in view.

III.—*Second Cycle.* This leads, normally, to the ***Baccalauréat,*** at the end of three years' study, in one of four different sections. The scheme will be clearer in tabular form.

<table>
<tr><th></th><th colspan="3">Pupils who learn Latin, with or without Greek.</th><th>Pupils who learn no dead language</th><th>Age from</th></tr>
<tr><td rowspan="4">PREMIER CYCLE.
4 years</td><td colspan="3">Sixième A (Latin).</td><td>Sixième B</td><td>10</td></tr>
<tr><td colspan="3">Cinquième A (Latin).</td><td>Cinquième B</td><td>11</td></tr>
<tr><td colspan="3">Quatrième A (Latin Greek)
Quatrième (Latin)</td><td>Quatrième B</td><td>12</td></tr>
<tr><td colspan="3">Troisième A (Latin Greek)
Troisième (Latin)</td><td>Troisième B</td><td>13</td></tr>
<tr><td></td><td>LATIN-GREC.</td><td>LATIN-LANGU.</td><td>LATIN-SCIENCES</td><td>Pupils who give up the study of Latin.
↓
SCIENCES-LANGUES</td><td></td></tr>
<tr><td rowspan="3">SECOND CYCLE
3 years</td><td>Second A</td><td>Second B</td><td>Second C</td><td>Second D</td><td>14</td></tr>
<tr><td>Première A</td><td>Première B</td><td>Première C</td><td>Première D</td><td>15</td></tr>
<tr><td>Philosophie A</td><td>Philosophie B</td><td>Mathématiques A</td><td>Mathématiques B</td><td>16</td></tr>
</table>

Let us now observe a little more closely just what is involved in this display, in the matter of studies and demands made upon the élève. As an important examination which we shall presently describe comes at the end of the *Première,* our present analysis will not pass beyond this grade. Here is the programme for a week.

There are several features of this scheme (we shall refer to

	VI. A.	VI. B.	V. A.	V. B.	IV. A.	IV. B.	III. A.	III. B.
French.	3	5	3	5	3	5	4	5
Latin.	7	..	7	..	6	..	6	..
Greek.	..	..	..	..	3*	..	3*	..
Modern Languages.	5	5	5	5	5	5	5	5
History and Geography.	3	3	3	3	3	3	3	3
Natural Science.	1	2	1	2	1	..	..	1
Physics and Chemistry.	..	..	..	..	..	2	..	2
Moral Philosophy.	..	..	..	..	1	1	1	1
Book-keeping.	..	..	..	..	..	1	..	1
Drawing.	2	2	2	2	2	2	2	2
Geometrical Drawing.	..	1	..	1	..	1	..	1
Arithmetic.	2	3	2	..	..	..	..	..
Arithmetic and Geometry.	..	..	..	3	2	3	3	..
Algebra and Geometry.	..	..	..	..	..	..	..	4
Writing.	..	1	..	1	..	..	..	..
Total No. Hours.	23	22	23	22	23	23	24	25

*Those who select Greek are exempted from 2 hours in modern languages and one in drawing.

	II. A.	II. B.	II. C.	II. D.	I. A.	I. B.	I. C.	I. D.
French.	4	4	4	4	4	4	4	4
Latin.	4	4	4	..	3	3	3	..
Greek.	5	..	..	..	5	..	..	..
Modern Languages.	2	7	2	7	2	7	2	7
Ancient History.	2	2	..	..	2	2	..	..
Modern History.	2	2	2	2	2	2	2	2
Geography.	1	1	1	1	1	1	1	1
Physics and Chemistry.	..	..	3	3	..	..	3	3
Practical Exs. in Science.	..	..	2	2	..	..	2	2
Geology.	**	**	**	**	..	..	..	..
Supplementary Exs. in Latin.	..	..	..	..	2	..	..	..
Drawing.	2	2	2	2	2+2†	2+2†	2	2
Geometrical Drawing.	..	..	2	2	..	..	2	2
Algebra and Geometry.	2	2	5	5	2	2	..	..
Geometry, Descriptive Geom. Trig., Algebra.	..	..	..	..	..	..	5	5
Total No. Hrs.	24	24	27	26	23	21	26	28

** 12 lectures of 1 hour.
†Optional.

four), which are particularly interesting.

1.—The prominence given to the study of French throughout.

2.—That all élèves at the age of 10 or 11 commence the study of m o d e r n languages (English, German, Italian, Spanish, Russian, or, in Algiers, Arabic), and continue it during six years at least, before matriculating into schools of university grade. Not only do they get glimpses of the best things in the literature of the language, but also learn to speak the language with considerable freedom and remarkable correctness in pronunciation. The direct method is employed and no word of French is ever spoken in the advanced classes. The majority of the élèves choose German, as this is required of all candidates for entrance into such military schools as the École Polytechnique and École de St. Cyr. On the other hand there is an increasing number taking up the study of English which is required for the École Navale.

3.—The proportion of recitation periods devoted to mathematics. As the drawing courses are about equally of a free hand and of a geometrical character, half the time given to drawing may be counted as mathematical and the percentages of recitation periods "required for mathematics during the four years of the Premier Cycle are: In the Latin course 14 per cent, 14 per cent,

22 per cent, 22 per cent; in the Modern Language course, 22.7 per cent, 22 per cent, 22 per cent and 22 per cent. In the Second and Première of the Second Cycle the percentages run: in the Latin-Greek course, 12.5 per cent, 4.5 per cent; in the Latin-Modern-Language course, 12.5 per cent, 6 per cent; in the Latin-Science course, 23 per cent, 24 per cent; and in the Modern-Language-Science course, 30 per cent, 30 per cent." To sum up from the Dixième to the Première the boy has spent 10.5 per cent, 11 per cent, 19.4 per cent, 22.8 per cent of his class hours in mathematical recitation according as he has pursued the courses leading to Première A, B, C or D. This emphasis which the French lay on mathematics is interesting and although the percentages may be somewhat higher than in America the training received is *vastly* superior in France. The fact that practically all the *professeurs titulaires* in the French lycées, even those in charge of the very elementary classes, are *agrégés* in the subjects which they teach means much. Just how much we shall explain later, but suffice it to remark here that no other country imposes as high standards for its professors of secondary education.

Another feature of mathematical instruction which is particularly interesting to us, is, that from the troisième on, that is, from the time the boy is 13 or 14 years old, instruction is usually given entirely by lecture. Indeed, even in classes before the troisième when a text-book is generally in the hands of the élève, he is required to take notes "pour préciser" the various topics. By such methods, searching questioning and frequent "tests," on the part of the professor, and rigid inspection, kindly expressed praise or cutting public reprimand on the part of the *proviseur* (director of the lycée), there is no possibility of learning parrot-fashion—no room for the shirker or the boy who does not try his best; reasoning powers and independence of thought must be constantly exercised. The élèves are encouraged to consult the various text-books to be found in all the lycée libraries and for those less bright this may be almost a necessity from time to time; but on personal inspection in different lycées I found the note books of élèves of 14 or 15 alike remarkable for their neatness and completeness. The habits thus gained in the lycée stand in good stead when the student reaches the university. The rapidity of the lecturer and the complexity of his theme seem to make little difference, for at the close of the hour the whole is in the note books as neat as copper-plate.

4.—The large number of hours in class recitation may not at first appear very imposing; but we cannot fail to be astonished that 8 hours per day (in class and in preparation of lessons) may be demanded from élèves in the premier cycle, and 10½ in summer, 10 in winter from those in the second cycle. The law further explicitly states that there is no limit to the number of hours which may be demanded of the élèves in the *Classe de Mathématiques Spéciales*. When we later come to look more closely at their programme we shall not be surprised, but nevertheless wonder, how these undoubtedly happy and healthy young men of 17 or 18 have survived the treatment. In more advanced lycée courses as well as at the universities I was also impressed with the almost appalling intensity and seriousness of the auditors —the strife is too strenuous, the competition too keen, to admit of a moment's levity or wandering thought. But when the lesson is over, every care is instantly banished and the national gaiety is once more in evidence.

To return to our table. We remark that the two groups of élèves who elect sciences on entering the Second Cycle have the same number of hours per week in mathematics—indeed the courses are identical. To give greater definiteness to our ideas as to their general attainments let us consider the programme of studies from Première D, when the boy is 15 or 16 years old.

French.—Lectures and questions on the principal French writers of the nineteenth century. Study of selections from prose writers and poets, from moralists, orators, politicians, scientists and historians of the sixteenth, seventeenth, eighteenth and nineteenth centuries.

History.—Political history of Europe in the eighteenth century. Detailed history of France at the close of the eighteenth century.

Geography.—Detailed study of France, its geological constitution, its climatology, physiography, topography, economic and military organization; its colonies, etc.

Physics.—Optics, electricity.

Chemistry.—Of the carbon compounds.

German.—Selections from the dramatic poetry of Schiller, Goethe, Kleist and Grillparzer. Extracts from the prose works of Wieland, Goethe, Schiller, Auerbach, Freytag, Scheffel, etc

English.—Shakespeare's Julius Cæsar and Macbeth, extracts from Milton, Addison, Goldsmith, Wordsworth, Byron, Coleridge, Dickens, Macauley, Eliot, Tennyson, and Thackeray.

Algebra.—Equations and trinomials of the second degree.

Calculation of the derivatives of simple functions; study of their variation and graphic representation; study of rectilinear motion by means of the theory of derivatives; velocity and acceleration; uniformly changing motion.

Geometry.—Solid.

Descriptive Geometry.—Elements.

Trigonometry.—Plane, including the use of four or five place logarithm tables, the solution of triangles and trigonometric equations.

Having finished the Première, the élève presents himself for examination under conditions which once more emphasize the unity of the French educational system. This is the examination for the first part of the state degree known as the *Baccalauréat.*

A peculiar feature of this examination is that it is not held in the lycées but at the university of the académie to which the particular lycée belongs.[4] As various civil and practically all government positions, except those in post and telegraph offices are only open to *bacheliers,* the state introduces into the body of examiners some who are wholly independent of the lycées. These examiners are the professors in the universities.

Since our future mathematicians are to come from Première C and D we shall give a few particulars concerning their examination. All examinations for the baccalauréat are held in July and October—at the ending of one school year and the beginning of the next. The examiners of the candidates from Première C are six in number, three of whom are university professors and three professors from the lycées or collèges; for Première D there are but two university professors in addition to three from the lycées. The examinations in all sections are both written and oral. Here is the scheme of examination which practically covers what the élève has studied in earlier years.

Première C (Latin-Sciences).—*Written.* 1st, a French composition (3 hours); (the candidate has a choice of three subjects); 2nd, a Latin translation (3 hours); 3rd, an examination in Mathematics and Physics (4 hours). *Oral* (about three-quarters of an hour). 1st, explanation of a Latin text; 2nd, explanation of a French text; 3rd, examination in a modern language—questions and answers being necessarily in this language. Questions in—4th, History; 5th, Geography; 6th, Mathematics; 7th, Physics; 8th, Chemistry.

[4]As there is no university at Chambéry, the candidate presents himself before a faculty of either Lyons or Grenoble.

And similarly for Première D.

The searching character of the tests prepares us for a large number of failures. Here is the record of the percentage of candidates passed in (1) July, (2) October, 1909: *Latin-Grec* (1) 44, (2) 42; *Latin-Langues Vivantes* (1) 41, (2) 42; *Latin-Sciences* (1) 49, (2) 46; *Sciences-Langues Vivantes* (1) 42, (2) 39. We observe that less than fifty per cent of the pupils get through on the first examination[5] while a similar percentage of the remainder fail and are required to return to the Première at once or wait for another year.[6] Those who have been successful return to the lycée to prepare for the second part of the baccalauréat. A choice of two courses (which may be slightly varied), is open to them, the one *Philosophie A* or *B*, the other, *Mathématiques A* or *B*. We shall only refer to the latter which has been supplied with pupils from the Première C and D. There they had 26 and 28 recitation hours per week. This has now been increased to $27\frac{1}{2}$ and $28\frac{1}{2}$. There has been an increase in the number of hours devoted to mathematics, physics and chemistry, but a reduction in the amount of study of modern languages. Latin no longer enters. The programme for Mathématiques A is in outline as follows:—

Philosophy (3 hours). *History and Geography* ($3\frac{1}{2}$ hours). *Modern Languages* (2 hours). *Physics and Chemistry* (5 hours). *Natural Science* (2 hours). *Practical Exercises in Science* (2 hours). *Drawing* (2 hours). *Hygiene* (12 lectures of 1 hour).

Mathematics. (8 hours): *Arithmetic.*—Properties of integers; fractions; decimals; square roots; greatest common divisors; theory of errors; etc.

Algebra.—Positive and negative numbers, quadratic equations (without the theory of imaginaries), progressions, logarithms, interest and annuities, graphs—derivatives of a sum, product, quotient, square root of a function, of sin x, cos x, tan x, cot x. Application to the study of the variation and the maxima and minima, of some simple functions, etc.

Trigonometry.—Circular functions, solution of triangles, applications of trigonometry to various questions relative to land surveying.

Geometry.—Translation, rotation, symmetry, homology and

[5]For some it may have been the third or fourth trial.

[6]There are certain exceptional cases which I shall not consider.

similitude, solids, areas, volumes, poles and polars, inversion, stereographic projection, central projections, etc.

Conics.—Ellipse, hyperbola, parabola, plane sections of a cone or cylinder of revolution, etc.

Descriptive Geometry.—Rabatments—application to distances and angles—projection of a circle—sphere, cone, cylinder, planes, sections, shadows—application to topographical maps, etc.

Kinematics.—Units of length and time. Rectilinear and curvilinear motion. Translation and rotation of a solid body. Geometric study of the helix, etc.

Dynamics and Statics.—Dynamics of a particle, forces applied to a solid body, simple machines in a state of repose and movement, etc.

Cosmography.—Celestial sphere, earth, sun, moon, planets, comets, stars—Co-ordinate Systems, Kepler's and Newton's laws, etc.

One of the most striking things in this scheme, as compared with American method, is to find arithmetic taught in the last year of the lycée course. Note, too, that from the Cinquième on, it has been taken up in connection with instruction in geometry and algebra. Indeed, this method of constantly showing the interdependence or interrelation of the various mathematical subjects was one of the interesting and valuable characteristics of French education as I observed it. For example, I happened to be present in a classroom when the theory and evaluation of repeating decimals was under discussion. After all the processes had been explained, problems which led similarly to the consideration of infinite series and limits were taken up. By suggestive questioning a pupil found the area under an arc of a semi-cubical parabola and the position of the centre of gravity of a spherical cap. With us it is not till the graduate school of the university that the boy is taught the true inwardness of such processes as long division and extraction of roots; but in France, arithmetic is taught as a science, not as an art, and the élève leaving the lycée has a comprehending and comprehensive grasp of all he has studied.

We remark that most of the mathematical subjects mentioned above are more or less foreign to our secondary education. Instruction in geometrical conics (*courbes usuelles*), is infrequently given by us, even in universities. Again, the ordinary mathemathical student who goes up for his doctor's degree in America may have the vaguest idea of what is even meant by Descriptive

Geometry. True, it is a regular course for our training of the engineer; but not, unfortunately, of the mathematician. On the other hand the French mathematical student has had at least four years of Descriptive Geometry, two of them before receiving his baccalauréat. The subject is required for admission into many government schools.

We note that the idea of a derivative is familiar to the lycéen during the last two years of his course. Why we so generally shut out the introduction of such an idea into our first courses in analytical geometry and theory of equations is, to me, a mystery. Finally, I would remark that the classes in Mathématiques A last two hours, with the exception of five minutes for recreation at the end of the first hour. The professor thus has sufficient time to amplify and impress his instruction.

At the close of the last year of the Second Cycle, the élève takes the examination for the second part of the baccalauréat. The same general conditions prevail as for the first part. The jury of four contains two university professors. The written examinations in mathematics, physics and philosophy are each three hours long; the oral covers what has been studied the year previously. If successful, a diploma now called the *baccalauréat de l'enseignement secondaire,* is granted to the élève by the Minister of Public Instruction. The élève thus becomes a *bachelier.* Diplomas in all four sections are of the same scholastic value. The charge made for diploma and examination is 90 francs. More than forty per cent of the candidates failed to pass at each of the examinations in 1909.

Because of the similarity of title used in the different countries, the Frenchman does not generally understand what the title Bachelor of Arts implies nor is it easy to make any concise statement in explanation. Little exaggeration can be made, however, in placing the bachelier on a plane of scholastic equality with the Sophomore who has finished his year at one of the best American universities.

Furthermore, his training has been undoubtedly much more thorough. After the age of 6 or 7 French boys are taught by men.[7] These men have all studied at the University and have passed the *examen de licence.* With very few exceptions now, the instructors have also passed the extremely difficult *Examen d' agrégation* in the subjects they propose to teach. By comparison, how woefully deficient our teachers of like grades! The recently

[7]Girls are taught by women. Coëducation does not exist in the lycées.

published reports of the United States sub-committees of the International Commission on the Teaching of Mathematics state the case frankly. That precious years are often lost to our youth by their inferior instruction is obvious to every one. As to Examinations—no guessing of possible questions and "cramming" for the same, so common in America, can qualify a student to pass an examination in France. The rigorous and impartial tests for promotion are conducted, at least in part, by those outside the lycée and pressure brought to bear upon teachers to promote ill-prepared pupils is unknown. According to a recent report of the Carnegie Foundation for the Advancement of Teaching, this is a "great source of weakness" and "a fruitful source of demoralization in American public schools."

I should now like to tell you something of the fourth section of lycée instruction, namely, *the Classes de Mathématiques Spéciales.*

If the bachelier who is proficient in mathematics be not turned aside by circumstances or inclination, to seek immediately a career in civil or government employment, he most probably proceeds to prepare himself for the highly special and exacting examination necessary for entrance into one of the great schools of the government. The method of this preparation exhibits a very peculiar feature of the French system. Whereas with us, or with the German, the boy who has finished his regular course in the secondary school goes directly to some department of a university for his next instruction, the bachelier, who has a perfect right to follow the same course, returns to his old lycée (or enrolls himself at one of the great Paris lycées, such as Saint Louis, Louis le Grand or Henri IV), to enter the *Classe de Mathématiques Spéciales préparatoire* which leads up to the *Classe de Mathématiques Spéciales.* The latter is exactly adapted to prepare students for the École Normale Supérieure, the École Polytechnique and the *bourses de licence.* Only a small proportion of the lycées (36 out of the 115), have this *Classe;* but with the exception of Aix they are to be found in all university towns. On the other hand, yet other lycées have classes which prepare specially for the less exacting mathematical entrance examinations of the École Centrale, École de Saint Cyr, École Navale, etc. But the number of élèves who on first starting out deliberately try to pass examinations for these schools is small, in proportion to the number who eventually reach them after repeated but vain effort to get into the École Polytechnique or the École Normale

Supérieure. Just what makes these two schools famous and peculiarly attractive will appear in a later section. It has been noticed that when the élève has won his baccalauréat he may immediately matriculate into a university, and although it might be possible for him to keep pace with the courses, in mathematics, at least, it would be a matter of excessive difficulty. There is then in reality, between the baccalauréat and the first courses of the universities, a distinct break, bridged only by the *Classes de Mathématiques Spéciales.*[8]

The élèves who enter the *préparatoire* section of this class are, generally, bacheliers leaving the classes de Mathématiques; in very rare instances, there are those who come from the classe de Philosophie. Natural science, history and geography, philosophy—indeed practically every study except those necessary for the end in view, have been dropped and from this time on to the agrégation and doctorat all energies are bent in the direction of intense specialization. This is the most pronounced characteristic of French education to-day. In mathematics, instruction now occupies 12 instead of 8 hours. New points of view, new topics and broader general principles are developed in algebra and analysis, trigonometry, analytical geometry and mechanics. Physics and chemistry are taught during six hours instead of five. Add to these, German, 2 hours; French literature, one hour; descriptive geometry, 4 hours; drawing, 4 hours. After one year of this preparatory training the élève passes into the remarkable Classe de Mathématiques Spéciales.

Eight years of strenuous training have made this class possible for the young man of 17 or 18 years of age, who is confronted with no less than 34 hours of class and laboratory work per week and no limit as to the number of hours expected in preparing for the classes!

When first I looked over the programme it seemed a well nigh impossible performance for one year. Surely no other country can show anything to compare with it.

Did time permit it would be interesting to reproduce in full the mathematical programme as given at the end of the *plan d'étude,* but I shall hastily refer to only a few of the subjects treated: In *Algebra and Analysis* we find developed, the fundamental ideas concerning irrational numbers, convergency and divergency

[8]It is only for mathematical or scientific students that such a break occurs, as no special classes are provided in other subjects except in the case of half a dozen Paris lycées which have classes in "letters" preparatory for entry into the Ecole Normale Supérieure.

of series, the elements of the theory of functions of a real variable, power series, their multiplication and division, their differentiation and integration term by term. Taylor's formula, the theory of algebraic equations, including symmetric functions, but omitting the discussion of infinite roots. The latter part of the course treats of differentials of several variables, elementary ideas concerning definite integrals, integration of such functions as are considered in a first calculus course of the best American colleges, rectification of curves, calculation of volumes, plane areas, moments of inertia, centres of gravity, differential equations of the first order, solutions of simpler differential equations of the second order, which occur in connection with problems of mechanics and physics. Whenever possible in the discussion of these topics the power to work numerical examples is emphasized.

Plane Trigonometry and the discussion of spherical trigonometry through the law of cosines are treated in class and five-place tables are used.

In the course on *Analytical Geometry* is given a thorough discussion of equations of the second degree, of homography and anharmonic ratios as they enter into the discussion of curves and surfaces of the second degree, of points at infinity, asymptotes, foci, trilinear coördinates, curvature, concavity and convexity, envelopes, evolutes. The professor also discusses thoroughly the various questions connected with the treatment of quadric surfaces and less completely, the theory of surfaces in general, of space curves, osculating planes, curvature of surfaces. The elements of the theory of unicursal curves and surfaces and of anallagmatic curves and surfaces are also taken up.

So also, we find broadly arranged programmes mapped out in mechanics and descriptive geometry. The whole number of class hours per week is broken up as follows:

Mathematics, 15; physics, 7 (2 in laboratory); chemistry, 2; descriptive geometry, 4; drawing, 4; German, 2; French, 1. The scope of the mathematical work may be judged from some books which were prepared with the needs of such a class especially in view.

B. Niewenglowski, Cours d' algèbre, I, 382 p.; II, 508 p.; Supplement—*G. Papelier* Précis de géométrie analytique, 696 p. —*Girod* Trigonométrie, 495 p.— *P. Appell* Cours de mécanique, 650 p.—*X. Antomari* Cours de géométrie descriptive, 619 p.

If anything, this list underestimates the work actually covered[9]

[9]That is, much more than what is called for by examination questions is studied. The élèves find truth in the adage: *Qui pent le plus pent le moins.*

by those who finally go out from the class. Tannery's Leçons d'algèbre et d'analyse (I, 423 p., II, 636 p.), might well replace Niewenglowski's work while Niewenglowski's Cours de géométrie analytique (I, 483 p.; II, 292 p.; III, 569 p.), represents the standard almost as nearly as Papelier's volume. Another treatise on mechanics widely used is that of Humbert and Antomari.

When we further realize that the main parts of the books in this list, which represents the work for only one of a half dozen courses, are covered by the professor in about fifteen months—the last three months of the second year are given over to drill in review and detail—we begin to get some conception of what the *Classe de Mathématiques Spéciales* really stands for. In his instruction the professor is officially "recommended" "not to overload the courses, to make considerable use of books, not to abuse general theories, to expound no theory without numerous applications dealt with in detail, to commence invariably with the more simple cases, those most easy to understand, for leading up finally to the general theorems. Among the applications of mathematical theory, those which present themselves in mathematical physics should be given the preference, those which the young people will meet later in the course of their studies either theoretical or practical. Thus in the construction of curves, choose as examples those curves which present themselves in Physics and Mechanics, as the curves of Van der Waals, the Cycloid, the Catenary, etc.—in the theory of envelopes choose those examples of envelopes which are met in the theory of cylindrical gearing—and so on. The pupils should be trained to reason directly on the particular cases and not to apply the formulæ. To sum up, one ought to develop their judgment and their initiative—not their memory."

(*Continued in February Issue.*)

AN ELEMENTARY EXPOSITION OF THE TIDES.

By Edison Pettit.
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In the study of the tides are found many interesting applications of the laws of gravitation and motion that appeal with peculiar fascination to the student of physical sciences. Indeed, to philosophers and scientists of all times they have presented an enigma of ever unfolding detail. Until the law of gravitation was entirely understood this puzzling phenomenon was the basis of many speculations. Plutarch, Pliny and Cleomedes declared that "the moon produced the tides," but this was far from demonstrated. On the other hand such noted investigators as Kepler and Galileo refused to believe that the moon had any relation to the tides, and it was not till Newton began and Laplace completed their mathematical demonstration that the true nature of this periodic movement of the ocean waters was recognized. The tides are caused by the sun and moon attracting various points on the earth with different intensities, such that those points more distant from these bodies are attracted the less. It is clear then that before we can explain tidal phenomena an understanding of the solar system and the laws governing it becomes necessary.

The Solar System.

Gravitation is a force that every particle of matter possesses in proportion to its mass to attract every other particle with an intensity that varies with the inverse square of the distance separating it from the particles considered. Put in symbol form this statement is

$$(1) \qquad i=c\frac{m}{r^2}.$$

where i is the intensity of the attraction of a particle whose mass is m at a distance of *r*, and *c* is a constant depending on the units employed necessary to render numerical results. From this the plain statement might be obtained that gravitation is a force that diminishes with the distance of a body and increases with the body's mass.

When several particles attract several others the two groups are found to attract each other as if the entire mass of each group were concentrated at a point in each which is called the center of gravity. This point is that which if an axis be

passed through it and the group of particles placed in any position the second group produces no tendency to rotate around in the first. Evidently then the center of gravity of a homogeneous

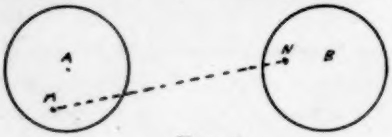


FIG. 1.

sphere is at its center. Consider the two spheres A and B of Figure 1. The particle M attracts the particle N, but the result of all the particles in A attracting all those in B is the same as that of a particle having the same mass as the sphere A placed at its center attracting all those in B concentrated in its center. It is well to note in this connection that A and B do not attract each *other* thus entirely from their centers, but only as if each were concentrated there.

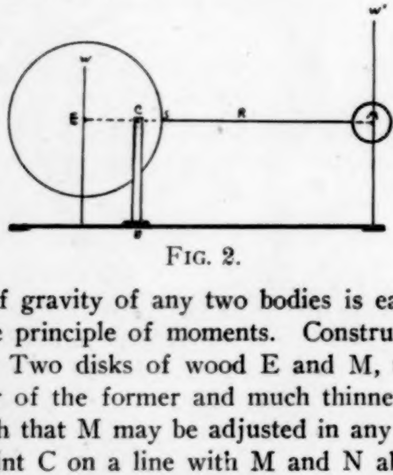


FIG. 2.

The center of gravity of any two bodies is easily determined by applying the principle of moments. Construct an apparatus like Figure 2. Two disks of wood E and M, the latter about ¼ the diameter of the former and much thinner are connected by a rod R such that M may be adjusted in any position. Support E at a point C on a line with M and N about ⅔ the distance from E to S. Studs at the centers of E and M serve to attach small weights. By adjusting M on R it may be arranged so that on moving M to any point in its orbit about C it will remain there. Upright wires W and W′ make this a very convenient apparatus for studying the tides. Let E and M represent the masses of the respective disks determined with a balance, then by varying these masses and adjusting the apparatus into equilibruim each time it may easily be shown that

$$E\ \overline{EC} = M\ \overline{CM} \qquad (2)$$

which is the law of moments. Notice here that with any three

of these quantities known the other may be obtained from this equation. Keeping these principles before us we will see how they apply to the solar system and the explanation of the tides.

In Figure 3 let S represent the sun, E the earth and M the moon. According to what has gone before S attracts E and M as if from the center of gravity of the earth and moon C, which of course lies between E and M. It follows then that it is this C and not the center of the earth E which moves on the nearly circular orbit about the sun. It will be of interest to know where this point is located. Let us apply our law of moments

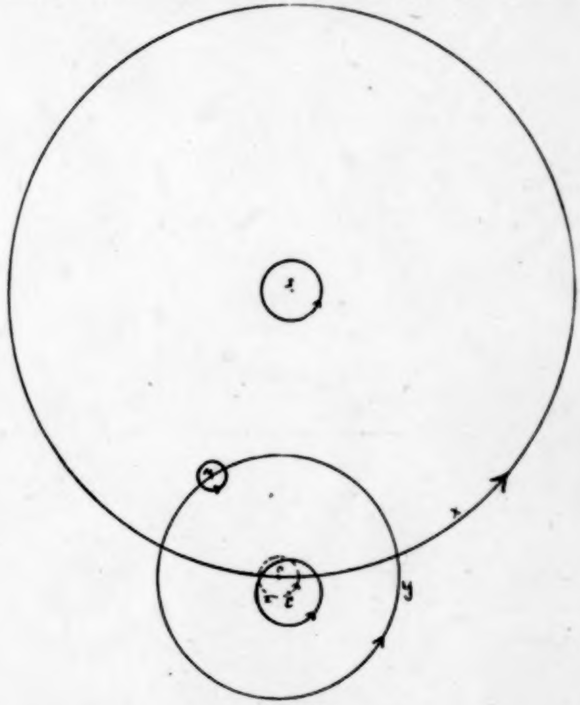


FIG. 3.

Various methods have shown that the earth is 81.5 times as heavy as the moon, i. e.,

(3) $$E=81.5\ M.$$

and the distance between their centers is 238,840 miles, i. e.,

(4) $$\overline{EC}+\overline{CM}=238840 \text{ mi},$$

(5) $$\text{or } \overline{CM}=238840 \text{ mi.}-\overline{EC}.$$

Substituting (3) and (5) in (2) we have

(6) $$81.5 \text{ m. } \overline{EC}=\text{M. } (238840-\overline{EC}) \text{ miles,}$$

$$\text{solving, } \overline{EC}=2895 \text{ mi.}$$

The equatorial radius of the earth is 3,963 miles, therefore the point C must be 1,068 miles below the earth's surface at a place where the moon is in the zenith. The ellipticity of the earth will slightly diminish this value when the moon is far north or south.

From this it follows that the moon is pulling the earth about in an orbit Z, 2,895 miles in radius every 27.3 days, and since it is the point C that moves on the earth's orbit and not the center E it is evident that as the earth revolves about the sun it must "flop over" 13 times, causing E to describe a wavy line. This motion of the earth about the small orbit Z causes an apparent displacement of the nearer planets among the stars, the measurement of which enables astronomers to comput the value of $\overline{EC}$ by the method of parallax. Placing this computed value of $\overline{EC}$ and in equation (2) the mass of the moon M may be found. This is one method of "weighing the moon," using nature's own balances, E C M.

Having examined the mechanism of the Solar System we are prepared to study the tides. Both sun and moon are found to cause the tides but it will be found convenient to introduce solar tides first.

Solar Tides.

Let C, Figure 4, represent the sun's position, A the earth and AA′ the earth's orbit. The earth here is vastly too big, for if it were drawn to scale having A C as the radius of its orbit, the earth A would only be .012 of a millimeter in diameter, and the variation from the true orbit, due to the "flopping" effect, would only be .0043 of a millimeter or about $\frac{1}{77}$ the width of this period. On the other hand, if A were maintained as it is, the sun C

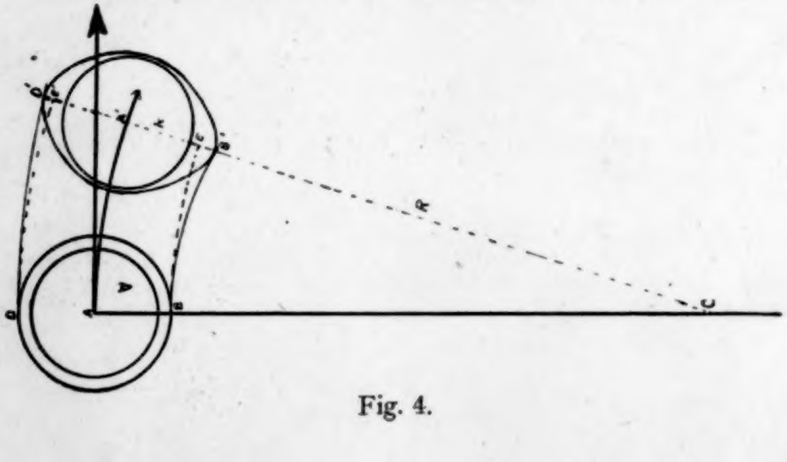


Fig. 4.

would be a great globe 11 feet in diameter at a distance of $\frac{5}{6}$ of a mile. We see then that this "flopping" effect of the moon on the earth will not hinder us in considering the earth's orbit as a true circle with the sun at the center. Further, owing to its minuteness, all points on the earth will describe practically the same orbit.

Suppose the earth surrounded by a thin shell of water DB to be at A. Now if the sun were not attracting it at C, the earth would move straight in the direction of the arrow. It will require some force to pull the earth out of this straight line and this the sun does by its attraction. The greater the force the sun exerts on A the more curved will this line be. Suppose a ball is tied to a string and the string fixed to a nail. If the ball is thrown it will move in a straight line until it stretches the string tight, when the string will begin to pull it in a circular orbit whose center is the nail. If a person standing at the nail should pull in on the cord the curvature of the ball's orbit would increase. We see then that the curvature of the orbit measures the force applied to the moving body. Thus if the intensity of the sun's attraction should be doubled the curvature of the earth's orbit would be doubled.

Now let the earth A start in the direction of the arrow, the sun pulls it into the curved orbit and A will arrive at A′. But the water at B just under the sun is attracted more than the earth A since it is closer, therefore, the sun will pull it more on one side, making its orbit more curved than $A\overline{S}$, and it will arrive at B′. Likewise the water at D opposite the sun will be attracted less than A because it is further away from C, consequently the sun will not pull it so far to one side as A, making the curvature of its orbit less than A and it will arrive at D′. It is easily seen then how the water becomes deeper at B′ and D′ and shallower at points 90° from these. This deepening of the water at B′ and D′ will continue until the attraction of the earth A′ overcomes this tendency of the water to leave its surface whence the tides remain as a bulge on the ocean. The diurnal rotation of the earth may bring a continent moving eastward to the tidal line B′A′D′ when the tide will run to a maximum and sink again on its eastern coast and be formed again on its western shore when the continent passes this line as we have already seen. From this it appears that a solar tide should arrive at a given place at noon and midnight with low water at 6 o'clock. Friction of the ocean bed, islands, etc., will delay the arrival of these tides as we

shall see in the discussion of Lunar Tides. At this point let us determine which of these tide waves B′ and D′ is the higher.

Height of Tidal Waves.

If the points B and D were rigidly attached to the earth and not mobile like the ocean, they would describe the orbits B E and D E′, respectively, concentric with A A′ about the sun C. The height of the tide wave at these points then is B′ E and D′ E′, respectively. But the orbits

$$(7) \qquad BE = DE' = AA'$$

nearly, both in curvature and length, since the earth's radius is so small ($\frac{1}{23250}$) in comparison with the radius of its orbit A′C. Therefore, $\overline{B'E}$ and $\overline{D'E'}$ are proportional to the differences of the curvatures of the orbits BB′, BE and DE′, DD′. Put in equation form this is

$$\frac{\overline{B'E}}{\overline{D'E'}} = \frac{\text{cur } BB' - \text{cur } BE}{\text{cur } DE' - \text{cur } DD'}.$$

From equation (7) we may replace BB′ and DD′ with AA′, therefore

$$\frac{\overline{B'E}}{\overline{D'E'}} = \frac{\text{cur } BB' - \text{cur } AA'}{\text{cur } AA' - \text{cur } DD'}.$$

It has already been shown that the curvature of our orbit is proportional to the attraction at that point, i. e., the curvature measures the intensity of the attraction. Then let i_0, i_1 and i_2 be the intensities of attraction of the sun on A′, B′ and D′, respectively, and since they are proportional they may replace cur AA′, cur. BB′ and cur. DD′, respectively, in the above equation, therefore

$$(8) \qquad \frac{\overline{B'E}}{\overline{D'E'}} = \frac{i_1 - i_0}{i_0 - i_2}.$$

By means of equation (1) we may obtain a numerical value for the left member of this equation.

Let K be the radius of the earth $\overline{A'B'}$, R the radius of the earth's orbit A′C and M the mass of the sun. Applying equation (1) to points A′, B′ and D′ we have

$$(9)\qquad \begin{cases} i_0 = c\dfrac{M}{R^2} \\ i_1 = c\dfrac{M}{(R-K)^2} \\ i_2 = c\dfrac{M}{(R+K)^2} \end{cases}$$

and subtracting

$$(10)\qquad \begin{cases} i_1 - i_0 = c\dfrac{M}{(R-K)^2} - c\dfrac{M}{R^2} = c\dfrac{MK}{R^2}\,\dfrac{2R-K}{(R-K)^2} \\ i_0 - i_2 = c\dfrac{M}{R^2} - c\dfrac{M}{(R+K)^2} = c\dfrac{MK}{R^2}\,\dfrac{2R+K}{(R+K)^2}. \end{cases}$$

Substituting these values in equation (8)

$$(11)\qquad \frac{\overline{B'E}}{\overline{D'E'}} = \frac{i_1 - i_0}{i_0 - i_2} = \frac{\dfrac{2R-K}{(R-K)^2}}{\dfrac{2R+K}{(R+K)^2}}.$$

Now R=92,900,000 miles and K=3,963 miles. Placing these values in (11)

$$\frac{\overline{B'E}}{\overline{D'E'}} = \frac{\dfrac{1}{45966778}}{\dfrac{1}{46503000}} = 1.0116$$

$$\text{or } \overline{B'E} = 1.0116\ \overline{D'E'} \text{ and } \overline{D'E'} = .988\ \overline{B'E}$$

from which it appears that the tide opposite the sun, $\overline{D'E'}$, is 98.8% as high as the tide under the sun, B'E.

In this connection we may inquire as to what the tide producing force of an attracting body like the sun or moon depends on. Referring to Figure 4 the height to which a tidal wave rises depends on the difference of the intensities of the gravitation of the sun at the center and surface of the earth. This is expressed by the equations in (10). Taking the first equation

$$i_1 - i_0 = c\frac{MK}{R^2}\,\frac{2R-K}{(R-K)^2}$$

it appears that relative to the value of R that of K is very insignificant, only $\frac{1}{23250}$, so that wherever it occurs except as a factor it may be thrown out of the equation. Eliminating K we have

$$(12)\qquad i_1 - i_0 = c\frac{2MK}{R^3}.$$

Here $i_1 - i_0$ is the tide raising force which we may donate by T, 2K is the diameter of the earth, M is the mass of the sun, R is its distance and c a constant necessary to obtain numerical results. This equation then may be extended to express the tide raising force of any attracting body. Let D be the diameter of the planet on which the tides are raised by the attracting body whose mass is M and distance R. Substituting these in equation (12).

$$T = c\frac{MD}{R^3} \tag{13}$$

From this equation we see that the tide producing force T and therefore the height of the tide varies directly as the product of the mass of the attracting body and the diameter of the planet and inversely as the cube of the distance of the attracting body. As to the amount of this tide raising force and its effects other than on the water we will discuss later.

Lunar Tides.

It has already been shown that tides on a planet are produced by the force of gravitation of an attracting body pulling the planet out of the straight line which by virtue of its inertia would otherwise be its orbit. We have seen how the moon is continually pulling the earth about in an orbit 2,895 miles in radius, therefore, the lunar tides will be formed in the same manner as the solar and the same explanation will satisfy both.

From the right member of equation (13) we see that the larger the planet, the greater the mass of the central attracting body and the less the distance between them the greater will be this tide raised on the planet. By far the most important of these quantities is R since it is cubed. The moon is much nearer the earth than the sun, hence although it is minute compared with the sun, yet the great proximity of our satellite may make the lunar tides even greater than the solar. To determine this point we shall compare the tide raising forces of the sun and moon as expressed in equation (13).

Let T = tidal force of the sun.
Let t = tidal force of the moon.
Let M = mass of the sun.
Let m = mass of the moon.
Let R = distance of the sun.
Let r = distance of the moon.
Let D = diameter of the earth.

Then by equation (13).

$$(14) \qquad \frac{T}{t} = \frac{c\frac{M\,D}{R^3}}{c\frac{m\,D}{r^3}} = \frac{M\,r^3}{m\,R^3}$$

The mass of the sun is 26,973,000 times the moon's mass and its distance 389 times; i. e., M=26,973,000 m. and R=389 r. Therefore substituting in (14).

$$\frac{T}{t} = \frac{26973000 r^3 m}{(389)^3 r^3 m} = .4582 \text{ or } 5/11 \text{ nearly.}$$

$$(15) \qquad \text{clearing } T = 5/11\ t.$$

But the ratio of the tide raising forces is the same as the ratio of the heights of the respective tides, just as the ratio of two pulls on a spring is the ratio of the lengths of the spring when these pulls are applied. Therefore, equation (15) means that the solar tide is only 5/11 as high as the lunar tide. Actual measurement shows this to be the case.

We may determine the relative heights of tides under and opposite the moon as we did in the case of solar tides by means of equation (11). Here $\overline{B'E}$ will be the height of the tide under the moon, $\overline{D'E'}$ the height of the tide opposite the moon, R the moon's distance and K the earth's radius. Now R=238,840 mi. and K=3,965 mi. Therefore

$$\frac{\overline{B'E}}{\overline{D'E'}} = \frac{\frac{2R-K}{(R-K)^2}}{\frac{2R+K}{(R+K)^2}} = \frac{\frac{8546}{10^{10}}}{\frac{8130}{10^{10}}}$$

Clearing,

$$(16) \qquad \overline{D'E'} = .9512\ B'E$$

that is, the tide opposite the moon is 95.12% as high as the tide under the moon.

Because of the uneven distribution of the oceans it is difficult to determine the height of a solar or lunar tide, but measures made on isolated islands show the moon's tide to be about 50 centimeters or about 20 inches, while the solar tide is 5/11 as great or 24 centimeters.

(*Continued in February Issue.*)

ARTICLES IN CURRENT PERIODICALS.

American Botanist for November: "Summer Flora of the Chicago Plain," Willard N. Clute; "The Glory of the Morning." Dr. W. W. Bailey; "Characteristics of Our Forest Trees"; "Trees that Yield Butter."

Catholic Educational Review for November: "The Cultural Aim Versus the Vocational," Thomas Edward Shields; "Specimen School Dramas," John Talbot Smith; "Higher Education for Catholic Women," Thomas C. Carrigan; "On the Management of Children Predisposed to Nervousness," Llewellys Barker.

Education for November: "The Function of a College Education," M. L. Crossley; "Moral Training in Public Schools," John T. Prince; "The Status of Japanese Students in America, Past and Present," Nenozo Utsurikawa; "Training for Social Efficiency—The Relation of Art, Industry and Education," Laura H. Wild; "The Ten Year Old Boy and His Books," M. A. Carringer; "The Cause, Cure and Prevention of Bad Habits," J. Mace Andress.

Educational Psychology for November: "An Investigation on the Value of Drill Work in the Fundamental Operations of Arithmetic. Part I," J. C. Brown; "Graded Mental Tests. Part III. Judgment, Conclusions and Summary of Results," Carrie Ransom Squire; "The Child's Speech. II. The Mother's Tongue," Robert MacDougall; "Writing Abilities of the Elementary and Grammar School Pupils of a City School System, Measured by the Ayres Scale," Irving King and Harry Johnson.

Mathematical Gazette for October: "The Theory of Proportion. (Continued.)," M. J. M. Hill; "Continuity of the Teaching of Mathematics from the Elementary Schools to the Secondary Schools," R. W. Jones; "The Teaching of Mathematics in Secondary Schools," W. J. Walker.

National Geographic Magazine for October: "The Wonderful Canals of China" (with 35 illustrations and 5 maps), F. H. King; "The Most Extraordinary City in the World" (with 60 illustrations), Shaoching H. Chuan, M. D.; "China's Treasures" (with 50 illustrations), Frederick McCormick.

Nature-Study Review for November: "The Interests of Children in Nature Material," Laura Emily Mau; "Collecting Things," Edwin E. Hand; "Observation Cages," Earl Lynd Johnston; "Birds and Deficient Children," Cyrus D. Mead; "School Gardening—Some Cautions," E. C. Bishop; "Hygiene as Nature Study. III. The Skin," F. M. Gregg.

Photo-Era for November: "The Two Great London Shows," A. H. Blake; "Winter-Activities," Virginia F. Clutton; "Independent Criticism," William H. Blacar; "A New Departure in Light and Shade Arrangements," Sadakichi Hartmann; "Photographing the Human Voice," Dr. A. Jencic; "The Photographic Picture-Postcard," James Thomson; "Color-Photography by Artificial Light," T. Thorne Baker.

Physical Review for October: "Ionization and Photo-electric Properties of Vapors of Alkali Metals," S. Herbert Anderson; "Heats of Dilution," William Francis Magie; "The Relation of Osmotic Pressure to Temperature," William Francis Magie; "On Characteristic Atomic Charges and Resultant Molecular Charges," Fernando Sanford; "Comparative Studies of Magnetic Phenomena, III. Magnetic Induction in a Group of Oblate Spheroids of Soft Iron," S. R. Williams; "Notes on Optical Constants of Metals," J. T. Littleton, Jr.; "The Kerr Rotation for Transverse Magnetic Fields, and the Experimental Verification of Wind's Magneto-Optic Theory," L. R. Ingersoll.

Popular Astronomy for December: "M. Henri Poincaré," F. R. Moulton; "A Brilliant Night Ahead," Frederick Campbell; "Three Interesting Spiral Nebulae," Heber D. Curtis; "Astronomy of Recreation," Alfred Rordame; "Note on the Binary Systems OΣ 536 and OΣ 341," R. G. Aitken; "Observations of Variable Stars Made at the Vassar College Observatory," Caroline E. Furness; "An Amateur's Observatory," Allan B. Burbeck.

Psychological Clinic for November: "Aspects of Infant and Child Orthogenesis," J. E. Wallace Wallin, Director of Psychological Clinic, School of Education, University of Pittsburgh, Pa.; "Are the Elementary Schools Getting a Square Deal?" G. W. Gayler, Superintendent of Schools, Canton, Ill.; "An Experiment in Concentration," Herbert F. Clark, Principal Olive Special School, Los Angeles, Calif.

School World for November: "The Psychology of Adult Reading," F. Smith; "The Methods of Teaching Reading in the Early Stages," Benjamin Dumville; "How Children Learn to Read," Barbara Foxley; "The Fifth International Congress of Mathematicians," P. Abbott; "The Second Moral Education Congress," Fred Charles; "Oxford Local Examinations, 1912," Hints from the Examiner's Report.

AN ANNOUNCEMENT CONCERNING THE AMERICAN MATHEMATICAL MONTHLY.

It will be of interest to many readers of SCHOOL SCIENCE AND MATHEMATICS to know that the *American Mathematical Monthly* is to be transferred in January, 1913, to the control of a Board of Editors representing nine universities and colleges which are contributing to a subsidy fund for its promotion.

One object in view is to provide a journal suited to the needs of teachers of college courses in mathematics and free scope will be given to contributions along historical and pedagogical lines. For instance, a prominent feature at the outset will be the serial publication of a recent research on "The History of the Logarithmic and Exponential Concepts" by Professor Florian Cajori of Colorado College, well-known as a writer on the history of mathematics. This work will be of intense interest to teachers of secondary mathematics as well as to those in the collegiate field.

A second object is to provide an opportunity for teachers who are not engaged in the more technical realms of mathematical research to cultivate the scientific spirit, especially through the reading of articles and discussions of moderate technical difficulty which stimulate the vision outward and upward. As is well-known, a teacher can never do his best work as a teacher without some such stimulus from without. Merely to gravitate about one's daily task on a dead level of mental activity offers little of inspiration or uplift; and yet the effort to reach to the very high levels is too great for the average teacher, the result being that mental effort to transcend one's environment struggles and dies unless there be within reach encouragement and steps up to the moderate levels.

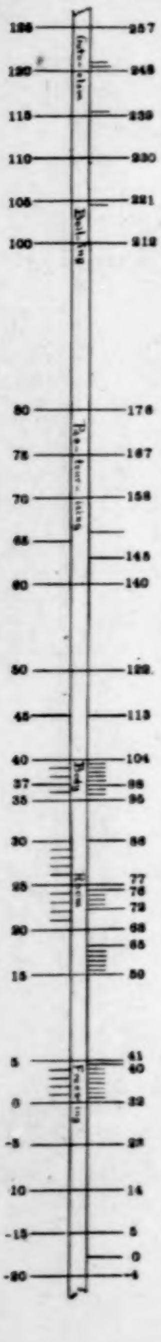
The *American Mathematical Monthly* has set for itself the task of providing such steps and encouragement for the great mass of mathematics teachers and especially for those teachers in the secondary schools who have a desire to keep up some contact with mathematics on the scientific side.

The subscription price of the *Monthly* will remain at the low figure of Two Dollars per year, notwithstanding its enlargement and the great addition of its cost of production.

Subscriptions should be made payable to *The American Mathematical Monthly*, and addressed to the Treasurer, B. F. Finkel, Springfield, Missouri.

Contributed articles and official correspondence should be addressed to the Managing Editor H. E. Slaught, 5548 Monroe Avenue, Chicago, Illinois.

C. F.



A LABORATORY THERMOMETER SCALE.

By Jean Broadhurst,
Teachers College, New York City.

The accompanying illustration shows a Centigrade-Fahrenheit thermometer scale designed for ready reference in the bacteriology, physical science, and domestic science classes at Teachers College. Centigrade and Fahrenheit equivalent are given for the temperatures most often needed by such students.

The scale is printed on narrow slips of rather stiff paper, and can be used as a book mark or pasted inside the cover of the laboratory book. The reverse side contains directions for changing from one scale to the other[1]: To convert degrees Fahrenheit into degrees Centigrade, subtract 32, multiply by 5, and divide by 9. To convert degrees Centigrade into degrees Fahrenheit, multiply by 9, divide by 5, and add 32.

Several high school text books in physics give two parallel thermometers. They are all drawn to a very small scale, and lack, so far as I know, any emphasis of the important temperatures. Stitt's text book in bacteriology (designed principally for medical students) gives several groups of equivalent temperatures, but drawn to varying scales. The markers illustrated above make no claim to originality it is hoped, however, that they may prove a handy laboratory aid.

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[1]The Editor of School Science and Mathematics prefers "the following method of making the transformations. Since —40° is the only temperature at which the temperature in either scale is represented by the same number, —40. C. or F., to change a Centigrade temperature (100C) into Fahrenheit, add 40°, multiply by 9, and divide by 5, after which subtract 40°. To change 248 F. into Centigrade, add 40°, multiply by 5, and divide by 9, after which subtract 40°. If your cut included —40° both methods could be taught from it."

PROBLEM DEPARTMENT.

By E. L. Brown,
Principal North Side High School, Denver, Colo.

Readers of this magazine are invited to send solutions of the problems in which they are interested. Problems and solutions will be duly credited to their authors. Address all communications to E. L. Brown, 3435 Alcott Street, Denver, Colo.

Algebra.

308. *Proposed by John M. Gallagher, Boston, Mass.*

In "Dubbs' Arithmetic Problems" occurs the following: "If the cost had been 8% less, the gain would have been 10% more. What was the per cent of gain?"

The solution is given as follows:

"Make 8 the first term, 10 the second term and 100—8 the third term of a proportion. The fourth term is the selling price.

Problem: Discover the general principle involved in the above method of solution; or, prove algebraically that the above solution is sound.

Solution by A. M. Harding, Fayetteville, Ark., and A. L. McCarty, Cape Girardeau, Mo.

Let s = selling price and c = the cost.

Let us make the problem general and write $a\%$ in place of 8% and $b\%$ in place of 10%.

The per cent of gain is $\frac{s-c}{c}\cdot 100$.

In the second case the cost is $\frac{(100-a)c}{100}$

And the gain $s-\frac{(100-a)c}{100}=s-c+\frac{ac}{100}$.

Hence the per cent of gain is

$$\frac{s-\frac{(100-a)c}{100}}{\frac{(100-a)c}{100}}\cdot 100=\frac{100s-(100-a)c}{(100-a)c}\cdot 100.$$

$$\therefore \frac{s-c}{c}\cdot 100+b=\frac{100s-(100-a)c}{(100-a)c}\cdot 100.$$

$$\text{or } \frac{a}{b}=\frac{100-a}{s}\cdot\frac{c}{100}.$$

Now if we put $c=100$ we have

$$\frac{a}{b}=\frac{100-a}{s}. \quad \text{(Dubbs' formula.)}$$

Since $s=\frac{(100-a)bc}{100a}$

and gain $= s - c = \frac{100(b-a)c-abc}{100a}$,

per cent gain $=\frac{s-c}{c}$. $100=\frac{100(b-a)-ab}{a}$. This is the general formula.

Special case—Put $a=8$, $b=10$:

then per cent gain is $\frac{100\ .\ 2-80}{8}=15$.

309. *Proposed by E. B. Escott, Ann Arbor, Mich.*

Solve:
$$x^2-yz=a \quad (1)$$
$$y^2-xz=b \quad (2)$$
$$z^2-xy=c \quad (3)$$

I. *Solution by Artemas Martin, Washington, D. C., and A. L. McCarty, Cape Girardeau, Mo.*

$(1)^2-(2)\times(3)$ gives
$$x(x^3+y^3+z^3-3xyz)=a^2-bc \quad (4);$$
$(2)^2-(3)\times(4)$ gives
$$y(x^3+y^3+z^3-3xyz)=b^2-ac \quad (5);$$
$(3)^2-(1)\times(2)$ gives
$$z(x^3+y^3+z^3-3xyz)=c^2-ab \quad (6).$$

From (4), (5), (6) we have
$$x^3+y^3+z^3-3xyz=\frac{a^2-bc}{x}=\frac{b^2-ac}{y}=\frac{c^2-ab}{z} \quad (7).$$

Therefore $y=\frac{(b^2-ac)x}{a^2-bc}$, $z=\frac{(c^2-ab)x}{a^2-bc}$.

Substituting in (1) and reducing,
$$x=\pm\frac{a^2-bc}{\sqrt{(a^3+b^3+c^3-3abc)}}.$$

Similarly, we find
$$y=\pm\frac{b^2-ac}{\sqrt{(a^3+b^3+c^3-3abc)}},$$
$$z=\pm\frac{c^2-ab}{\sqrt{(a^3+b^3+c^3-3abc)}}.$$

II. *Solution by Julia M. Bligh, Batavia, N. Y., and Eugene M. Dow, Brighton, Mass.*

Subtract (2) from (1) we get $(x-y)(x+y+z)=(a-b)$ (3).

Subtract (3) from (2) we get $(y-z)(x+y+z)=(b-c)$ (4).

Divide (3) by (4) we get $y=\frac{x(b-c)+z(a-b)}{a-c}$ (5).

Substituting (5) in (1) and (3) we get, after solving,
$$(ab-c^2)x^2+xz(a^2-ab-bc+c^2)-z^2(a^2-bc)=0.$$

whence $x=z$ or $x=\frac{bc-a^2}{ab-c^2}z$ (6).

If $x=z=y$ then $a=b=c=0$.

When $x=\frac{bc-a^2}{ab-c^2}z$, $y=\frac{ac-b^2}{ab-c^2}z$.

Substituting these values of x and y in (3) we get

$$z = \pm \frac{ab-c^2}{\sqrt{a^3+b^3+c^3-3abc}}.$$

whence

$$y = \pm \frac{bc-a^2}{\sqrt{a^3+b^3+c^3-3abc}}.$$

$$x = \pm \frac{ac-b^2}{\sqrt{a^3+b^3+c^3-3abc}}.$$

If $a = b = c$ then x, y and z are indeterminate.

III. *Solution by the Proposer.*

Multiply equations by y, z, x respectively, and add.

Also multiply by z, x and y and add.

We get $$ay + bz + cx = 0$$

and $$az + bx + cy = 0.$$

Solving these equations, we have

$$\frac{x}{a^2-bc} = \frac{y}{b^2-ca} = \frac{z}{c^2-ab} = n, \text{ say.}$$

Substituting
$$x = (a^2-bc)n,$$
$$y = (b^2-ca)n,$$
$$z = (c^2-ab)n,$$

in equation (1), we get for n,

$$n = \pm(a^3+b^3+c^3-3abc)^{-\frac{1}{2}}.$$

$$\therefore \quad x = \pm(a^2-bc)(a^3+b^3+c^3-3abc)^{-\frac{1}{2}}.$$

$$y = \pm(b^2-ac)(a^3+b^3+c^3-3abc)^{-\frac{1}{2}}.$$

$$z = \pm(c^2-ab)(a^3+b^3+c^3-3abc)^{-\frac{1}{2}}.$$

Geometry.

310. *Proposed by Benjamin E. Chiu, Shanghai, China.*

Divide a rectangle 7 inches long and 3 inches broad into three figures which can be joined together so as to form a square. (Ex. 28, p. 465, Phillips and Fisher's Elements of Geometry.)

I. *Solution by I. L. Winckler, Cleveland, Ohio.*

Let EBGF be the given rectangle, EB = 7 and BG = 3. Prolong EB to L, making BL = BG. On EL as a diameter describe a semi-circle. At B draw BC perpendicular to EL meeting the semi-circumference at C. Draw EC, meeting FG at H. On BC as a side construct the square BCDA, A being on EB between E and B, and AD meeting EC at K. Then the figures BGHKA, AEK, and EFH may be placed so as to form the required square.

For, let EB = b and BG = a.

Then $\frac{b}{BC} = \frac{BC}{a}$ (1) and from similar triagles EAK and EBC we have

$$\frac{b}{BC} = \frac{EA}{KA} \ (2)$$

From (1) and (2) $\frac{BC}{a} = \frac{EA}{KA}$ or $\frac{BC}{a} = \frac{b-BC}{KA}$ (3)

From (3) $\frac{b}{a+\text{KA}} = \frac{\text{BC}}{a}$ or $ab = \text{BC}\ (a+\text{KA})$ (4)

Since $ab = \overline{\text{BC}}^2$, (4) becomes $\text{BC} = a + \text{KA}$

or $a + \text{CG} = a + \text{KA}$ $\therefore$ KA = CG.

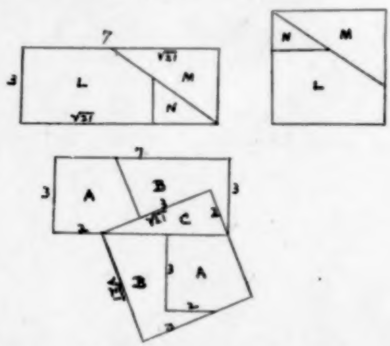
$\therefore$ Δ EAK = Δ HGC.

Also AD — KA = BC — CG $\therefore$ DK = BG = EF.

$\therefore$ Δ EFH = Δ DKC.

$\therefore$ EAK may be placed in position of CHG and EFH in position of DKC to form square ABCD.

II. *Solutions by E. B. Escott, Ann Arbor, Mich.*



Note.—The piece B must be turned over in making the square.

311. *Proposed by W. T. Harlow, Portland, Ore.*

Three men undertake to saw a log which is 4 feet in diameter. How far into the log should each of the first two saw in order to equalize the work?

I. *Solution by H. H. Seidell, St. Louis, Mo., and Walter C. Eells, Tacoma, Wash.*

If we assume that the work done is proportional to the area of the cross section that has been cut by the saw, then the problem is simply to find the central angle subtended by a segment whose area equals one-third the area of the circle.

If θ = the central angle we have

$$\tfrac{1}{2}r^2\theta - \tfrac{1}{2}r^2\sin\theta = \tfrac{1}{3}\pi r^2 \text{ or } \theta - \sin\theta = 2\pi/3 = 2.0944.$$

If we solve this equation by trial we find $\theta = 149^\circ\ 17'\ 40''$ nearly.

Now if a = distance from center to chord of segment we have $a = r \cos \tfrac{1}{2}\theta = 2 \cos 74^\circ\ 38'\ 50'' = 2 \times 0.26476 = 0.52992$ feet.

A — 0.52992 = 1.47008 ft. = 1 ft. 5.6 in.

Hence first man should saw a distance equal 1ft. 5.6 in.
Second man should saw a distance equal 1 ft. 0.8 in.
Third man should saw a distance equal 1 ft. 5.6 in.

II. *Solution by Nelson L. Roray, Metuchen, N. J.*

Let x = number of feet the first man saws.

Then $2\sqrt{4x - x^2}$ = number of feet in chord of arc first man saws.

Also $2\sqrt{x}$ = number of feet in chord of half arc first man saws.

$\therefore$ Length of this arc by Huygens Formula is $\dfrac{16x-2\sqrt{4x-x^2}}{3}$

The equation for the area of the sector sawed by the first man is easily shown to be

$$16\sqrt{x}-(8-3x)\sqrt{4x-x^2}=4\pi.$$

Whence $x = 1.47$ ft.

$\therefore$ First and third man each saw 1.47 ft. and the second man saws 1.06 ft.

312. *Proposed by Elmer Schuyler, Brooklyn, N. Y.*

In a given circle, inscribe a triangle, knowing the mid-points of the three minor arcs subtended by the three sides of the triangle. (From Ghersi.)

I. *Solution by A. M. Harding, Fayetteville, Arkansas.*

Let A′, B′, C′ be the given mid-points. Connect these points, forming triangle A′B′C′. Draw the lines from vertices of this triangle perpendicular to the opposite sides meeting the circle in A, B, C. Then ABC is the required triangle.

The proof easily follows.

Let O be orthocenter of ΔA′B′C′ and D, E, F the feet of the perpendiculars from A′, B′, C′ on B′C′, C′A′, A′B′ respectively.

Then ΔA′OF is similar to ΔC′OD.

$\therefore$ $\angle$FA′O = $\angle$DC′O. Hence arc CB′ = arc AB′ and B′ is mid-point of arc CA.

Likewise C′ is mid-point of arc AB.

And A′ is mid-point of arc BC.

II. *Solution by I. L. Winckler, Cleveland, O., and G. A. Van Derhule, Suffield, Conn.*

Let D, E and F be the given mid-points of the minor arcs subtended by the sides AC, AB and CB, respectively, and let O be the center of the given circle.

Since OD, OE and OF, are perpendicular to AC, AB and CB, respectively, angles DOE, EOF and DOF are supplements of angles A, B and C respectively,

Also angles A, B and C are supplements of angles BFC, ADC and AEB, respectively.

$\therefore$ Angles DOE, EOF and DOF are equal, respectively, to angles BFC, ADC and AEB.

$\therefore$ At E construct angles OEB and OEA equal to one-half angle DOF. Draw AB.

Then draw AC perpendicular to OD and draw CB. ABC is the required triangle.

CREDIT FOR SOLUTIONS.

303, 304, 305, 306, 307. Nelson L. Roray. (5)

305. L. S. Stephens. (1)

308. Eugene M. Dow, John M. Gallagher, A. M. Harding, A. L. McCarty, H. H. Seidell, Nelson L. Roray, I. L. Winckler. (7)

309. Julia M. Bligh, Eugene M. Dow, E. B. Escott, Artemas Martin, A. L. McCarty, Otto Ramler, Nelson L. Roray, Elmer Schuyler, I. L. Winckler. (9)

310. Norman Anning, E. G. Berger, E. B. Escott (2 solutions), Nelson L. Roray, H. H. Seidell, I. L. Winckler. (7)

311. Walter C. Eells, A. M. Harding, Nelson L. Roray, H. H. Seidell, H. E. Trefethen. (5)

312. Julia M. Bligh, Eugene M. Dow, A. M. Harding, Geo. R. Livingston, Otto J. Ramler, Elmer Schuyler, Nelson L. Roray, H. H. Seidell, Levi S. Shively, L. I. Thayer, G. A. Van Derhule, I. L. Winckler. (12)

Total number of solutions, 46.

PROBLEMS FOR SOLUTION.

Algebra.

323. *Proposed by Nelson L. Roray, Metuchen, N. J.*

In a series of equal ratios the sum of the mean proportionals of each antecedent to its consequent is equal to the mean proportional between the sum of the antecedents and the sum of the consequents.

324. *Proposed by Otto J. Ramler, Buffalo, N. Y.*

If ax^2+bx+c and $a'x^2+b'x+c'$ have a common factor of the form $x+f$, prove that

$$(ac'-a'c)^2=(bc'-b'c)\ (ab'-a'b).$$

Geometry.

325. *Proposed by Levi S. Shively, Mount Morris, Ill.*

Construct a triangle, given its circumcenter, its orthocenter, and the mid-point of one of its medians.

326. *Proposed by A. Babbitt, State College, Pa.*

In the triangle ABC find a point O such that the product of the perpendiculars from it upon the sides shall be a maximum.

327. *Proposed by H. E. Trefethen, Waterville, Me.*

In any given triangle ABC:

(a) Find a point O such that its distances from the sides are proportional to these sides respectively;

(b) Through this point O draw parallels EF to AB, GH to AC, IK to BC, and prove that the points E, G, I, F, H, K are concyclic;

(c) Prove also HF : EK : GI $=a^3:b^3:c^3$.

SCIENCE QUESTIONS.

BY FRANKLIN T. JONES,
University School, Cleveland, Ohio.

Readers of SCHOOL SCIENCE AND MATHEMATICS *are invited to propose questions for solution—scientific or pedagogical—and to answer questions proposed by others or by themselves Kindly address all communications to Franklin T. Jones, University School, Cleveland, Ohio.*

Questions and Problems for Solution.

100. *Proposed by C. A. Perrigo, Dodge, Neb.*

The ratio of masses of moon and earth is .0125 and ratio of their diameters is .273. With what acceleration would a body fall at moon's surface?

[From Hastings & Beach—General Physics, p. 67.]

84. How many grams of sodium will be needed to combine exactly with a quantity of chlorine which, at 18° C. and 740 mm. occupied a volume of 200 cc.? What will be the weight of the resulting salt?

200 cc. chlorine at 18° C. and 740 mm. occupies 182.7 cc.

1 liter of chlorine weighs 3.19 gm.; .1827 l. weighs .583 gm.

Reaction	Sodium	+	Chlorine	$\rightarrow$	Sodium Chloride
Equation	Na	+	Cl	$\rightarrow$	NaCl
Weights	23		35.5		58.5
	x gm.		.583 gm.		y gm.

$x = 23 \times .583 \div 35.5 = .377$ gm. sodium.

$y = 58.5 \times .583 \div 35.5 = .961$ gm. sodium chloride.

85. A cord is tied to two trees 8 feet apart. Suspended from the middle point of the cord is a camp kettle weighing 20 lbs., the point of suspension being 3 feet lower than the ends of the cord. What is the tensile force in the cord?

3 : 20 = 5 : force in cord.

Force in cord = $33\frac{1}{3}$ lbs.

87. What should be the level of water in a stand pipe to maintain a pressure of 70 pounds to the square inch, assuming the density of the water to be 62.5 lbs per cubic foot?

70 lbs per sq. in. = 70 x 144 = 9080 lbs. per sq. ft.

A column of water 1 foot high and 1 sq. ft. in cross-section weighs 62.5 lbs. and exerts a pressure of 62.5 lbs. per sq. ft.

Then $\frac{9080}{62.5} = 145.3$ feet, the height of the required water column.

88. Suppose that heat is supplied at a uniform rate to a mass of ice at —20° C. until it becomes steam at 120° C. Describe the resulting physical changes and give a rough estimate of the duration of each change compared to the time required to heat the freezing water to the boiling point. (Specific heat of ice 0.5; specific heat of steam 0.48.)

The changes produced are as follows:

(1) Warming the ice to 0° C.; (2) melting the ice; (3) warming the melted ice to 100° C.; (4) vaporizing the water; (5) warming the vapor to 120° C.

Assume 1 gram of ice as the mass of the ice.

Heat to warm ice to 0° = 20 x .5 x 1 = 10 cal.

Heat to melt ice = 1 x 80 = 80 cal.

Heat to warm to 100° = 1 x 100 = 100 cal.

Heat to vaporize = 1 x 536 = 536 cal.

Heat to warm steam to 120° = 20 x .48 x 1 = 9.60 cal.

Since the heat is applied uniformly, the times required are in the same ratio as the amounts of heat consumed, or times are in the ratios 10 : 80 : 100 : 536 : 9.60.

89. Find the volume in cubic centimeters of hydrochloric acid solution of 1.1 specific gravity, containing 20% of actual acid by weight, which will react with 10g of calcium carbonate.

Reaction	Calcium Carbonate	+	Hydrochloric Acid	$\rightarrow$	Calcium Chloride	+	Carbon Dioxide	+	Water
Equation	$CaCo_3$	+	2HCL	$\rightarrow$	$CaCl_2$	+	CO_2	+	H_2O
Weights	40.		2.						
	12.		71.						
	48.								
	100		73						
Combining Weights	10		x=7.3 grams of actual acid.						

7.3 gm. is 20% of whole, hence total weight of dilute acid = 36.5 gm.

The specific gravity of this solution is 1.1 or 1 cc. Solution weighs 1.1 gram. Then 36.5 ÷ 1.1 = 33.2 cc. is volume of acid solution required.

90. What volume of oxygen would be required to burn 1000 cc. of carbon monoxide, and what would be the volume of the product, all the gases being at the same temperature and pressure?

Reaction	Carbon Monoxide	+	Oxygen	$\rightarrow$	Carbon Dioxide
Equation	2CO	+	O_2	$\rightarrow$	$2CO_2$
Molecules	2		1		2
Volumes	2		1		2
	1000 cc.		x=500 cc.		y=1000 cc.

The volume of oxygen is x = 500 cc. of carbon dioxide; y = 1000 cc.

91. A gas main runs up a hill 100 meters high When no gas is flowing, the pressure in the main at the bottom of the hill is 8 grams per square centimeter in excess of the pressure of the surrounding air. What would be the difference between the pressure in the main and that of the surrounding air at the top of the hill under the same conditions? Assume that the average densities of the two columns are as follows: air, 0.0012 grams per cubic centimeter; gas, 0.0003 grams per cubic centimeter.

The column of gas is 100 m. = 10,000 cm. high. The volume for a cross-section of 1 sq. cm. = 10,000 cc.

10,000 cc. of air weighs 10,000 x .0012 = 12 gm.

10,000 cc. of gas weighs 10,000 x .0003 = 3 gm.

Assume the air pressure at the foot of the hill to be 1000 grams per sq. cm. Then the gas pressure, being 8 more is 1008. At the top of the hill the air pressure is 12 less, or 988. The gas pressure is 3 less or 1005. The difference (1005 — 988 = 17) is the difference in pressure in grams per sq. cm.

92. About 100,000,000 tons of water go over Niagara Falls every hour, and drop 161 feet. If all this water were run through a power house of 75 per cent efficiency, how many horse-power would be available?

100,000,000 tons = 10^8 tons = 2000 x 10^8 lbs.

2000 x 10^8 x 161 = 322,000 x 10^8 foot-pounds the amount of actual energy expended per hour. With an efficiency of 75 per cent, the amount of useful energy is 322,000 x 10^8 x .75 = 241,500 x 10^8 ft. lb. The energy per minute is 241,500 x 10^8 ÷ 60 = 4025 x 10^8 ft. lbs. The horsepower is 4025 x 10^8 ÷ 33,000 = 12,196,969.

93. Two kilograms of food at 90° C. (specific heat 0.5) are put into a refrigerator and cooled to 10° C. all of the heat going into the ice. How much ice is melted, if the water formed escapes at once?

2 kgm. = 2000 gm.

Heat from 2000 gm. (sp. ht. 0.5) through 80° C. = 2000 x .5 x 80 = 80,000 calories. To melt one gram of ice requires about 80 calories. Hence 1000 gm. of ice will be melted.

94. A "hylo" incandescent electric light bulb is provided with a switch by which the electric current can be sent through a low resistance filament for giving a bright light, or through a high resistance filament for giving a feeble light. The lamp operates on a 110-volt circuit, and the resistances of the two filaments are 220 ohms and 1,100 ohms respectively. If the cost for electric power is half a cent per hour when the low resistance filament is used, what is the cost per hour when the high resistance filament is used, and what is the rate per kilowatt-hour that is being paid for power? The phrase "per kilowatt-hour" means "per hour for each 1000 watts taken."

Amperes in low resistance filament = 110 ÷ 220 = .5.

Watts = 110 x .5 = 55 = .055 kilowatt.

$\frac{1}{2}$ cents per hour for 55 watts means 1000 x $\frac{1}{2}$ ÷ 55 = $9\frac{1}{11}$ cents per 1 kilowatt.

Amperes in high resistance filament = 110 ÷ 1100 = .1.

Watts = 110 x .1 = 11 = .011 kilowatt.

Cost per hour = .011 x $9\frac{1}{11}$ = $\frac{1}{10}$ cent.

95. Five grams of silver chloride can be obtained by adding silver nitrate to a certain amount of crystallized calcium chloride, $CaCl_2 6H_2O$. What is the weight of the calcium chloride?

Reaction	Silver Nitrate	+	Calcium Chloride	→	Silver Chloride	+	Calcium Nitrate	+	Water
Equation	$2AgNO_3$	+	$CaCl_2 6H_2O$	→	$2AgCl$	+	$Ca(NO_3)_2$	+	$6H_2O$
Weights			40		216				
			71		71				
			108						
			219		287				
			xgm.		5gm				

x = 219 x 5 ÷ 287 = 3.82 gm.

THE TABOO ON MODERATE DRINKING.

The use of alcohol is receiving some hard knocks these days. A prominent railway system, not content with the general rule heretofore in force on railways forbidding employees to drink while on duty, now forbids employees to indulge at all in drinking out of employment hours, or in any other conduct which will impair their health or make them less alert and less capable while on duty. The owner of one of the nation's pets—a prominent baseball team—announces that moderation in drinking is not sufficient; the players on his team must leave alcohol entirely alone and abandon cigarets. The justification for such rules may be found not only in the difficulty of being moderate in indulgence, but also in the cumulative and after-effects of dissipation. The world is moving; the old fetich of "personal liberty" at whatever cost of danger to the public at large seems to be losing its power. *The Journal of the American Medical Association* thinks that the time may come when every man to whom the life and safety of others are entrusted may be expected or even required to be as abstemious as ball-players and railway employees.

A TRAVELING UNIVERSITY.

A section of the University of Minnesota will go "on tour" again this year. For one week last June the people of a number of small towns in Minnesota had the State University in their midst in the form of its most characteristic activities, and the eighteen communities benefited have unanimously asked that the experience be repeated this year.

The plan is something more than merely university extension. To all intents and purposes a representative portion of the university—faculty, students, and equipment—is temporarily detached and transferred to other parts of the State, thus actually extending the benefits of the State's costliest educational plant to a wider field than ever before. The plan is considered by the United States Bureau of Education an excellent device for bringing together for mutual profit a State university and the people who support it.

What "University Week" really is may be seen from a typical program. Each day of the six is devoted to some special topic, with lectures and demonstrations during the daytime and high-class entertainments at night. Thus: Monday is BUSINESS MEN'S DAY. There are lectures on all kinds of topics interesting to business men, from marketing problems to fighting forest fires, as well as a few talks of more general nature. In the evening there is a concert by the University Glee Club. Tuesday is ART AND LITERATURE DAY, with lectures on libraries, children's books, women's clubs, civic betterment, the drama, and similar subjects. There is a reading hour in the afternoon, in charge of a trained elocutionist, and an industrial art exhibit; in the evening an illustrated lecture: "Art in Common Things."

Wednesday is HOME WELFARE DAY. In the day sessions such problems as "The Human Beings of High School Age," "Why Babies Die," rational living, kindergartens, and industrial education are considered, while at night a prominent educator gives an illustrated lecture on "How Minnesota Educates Her Children." Thursday is PUBLIC HEALTH DAY, with appropriate lectures and exhibits. In the evening there is a dramatic recital of a modern play.

Friday is FARMERS' DAY, and live questions of farm policy are discussed by experts in agriculture. There is also an address on "The Social Possibilities of Rural Communities" by an educator who has made special studies in this field. In the evening professors from the university give a scientific demonstration of the gyroscope and liquid air. Saturday is TOWN AND COUNTRY DAY, with "Social Life in Town and Country" as the leading topic. In the evening the University Dramatic Club appears in Shakespeare's Merchant of Venice.

Genuine interest is aroused in the towns visited. In most instances the people take the visit of the university as the business of the week, and devote all their attention to it. Not only the townspeople, but farmers from outlying districts as well, attend the sessions. Boy's farming camps are organized in connection with the University Week, and always prove a popular feature. The university authorities and those who co-operate with them—State health boards and other agencies—are particularly careful to provide speakers who not only know their subject well, but are able to talk interestingly to a nonuniversity audience. The expense of obtaining such men would be prohibitive but for an ingenious arrangement of circuits, whereby the traveling University Week is able to "play" six communities in the same neighborhood by interchanging days.

It is estimated that there are several ways in which this novel plan of extending the influence of a State university will have a direct effect:

In the first place, it will make the work of the university well and favorably known where it has before been known only vaguely or even mistakenly; It will break down the already weakening barrier of educational exclusiveness; and more important still, it is one more link in the chain of rural betterment. It emphasizes an essential point in the present-day conception of rural life—that town and country are one community.

NATIONAL EDUCATION ASSOCIATION.

Practically all of the chairmen of departments of the National Education Association met at Chicago, Oct. 23, in response to the call of Pres. E. T. Fairchild. At that conference, the program for the meeting to be held at Salt Lake City next July was blocked out in a large way. It was agreed that the major theme which should find expression throughout the general programs of the Association, and in the programs of each department, was to be the betterment of rural and elementary schools. It was understood that the natural function of no department should be impaired, that each department should do its distinct work, but that in its program it should relate the work of the department as far as possible to common life through rural and elementary education. The officers and heads of departments were practically united on the point that there should be a great central binding thread running through all the work of the Association, not, however, to exclude in any way the special work for which each department was created. President Fairchild expressed himself as being highly pleased with the attitude of the chairmen of the departments in their willingness to follow out his suggestion that the Association should attempt to meet the specific needs of all the teachers of all the schools.

WANTED—OLD TEXTBOOKS.

The government wants gifts of old or rare textbooks—at least, the government Bureau of Education does. Government bureaus are so in the habit of giving away documents instead of receiving them that a request like this warrants attention.

The point is that the United States Bureau of Education is endeavoring to get together the finest possible collection of textbooks in English, French, German, Spanish, Italian, Scandinavian, Dutch, and Greek languages published within the last two centuries, and hopes that possibly some of the many educators and investigators who have been recipients of the government's bounty by receiving valuable documents in the past may return the compliment now with an occasional textbook of by-gone days. An antiquated speller or a musty Xenophon may be just the book needed to fill an important gap in textbook history. The Library of Congress is aiding in the task, and the Bureau would appreciate gifts from individuals as well. "When this library is complete," says Commissioner Claxton, "It should become the Mecca of all students of this phase of education."

In the meantime, the Bureau continues to be generous in its opportunities to investigators. It now has one of the largest and most complete libraries of education in the world, containing about 70,000 bound volumes and 80,000 or more reports, pamphlets, and periodicals. Almost any of these books not obtainable in ordinary libraries may be secured by teachers and bona fide students of education, either through the local library or directly from the Bureau of Education, under certain conditions. Requests should be made to the librarian.

Another important assistance rendered by the Bureau is to commissions investigating particular phases of education. Without cost other than that of coming to Washington, representatives of these commissions may find practically everything that is now in print in pamphlets, books, or magazines on any subject of education, including educational legislation, frequently accomplishing in a few days or weeks what would otherwise take months. The Bureau of Education thus fulfills a peculiar public service in acting as a source of educational information, and real seekers after knowledge will find every aid and encouragement given them by those in charge.

PLATINUM HAS MANY USES.

The mineral called platinum is really a natural alloy of platinum, iridium, rhodium, palladium, and often osmium, with varying amounts of iron, copper, and gold, according to the United States Geological Survey. It is usually found as small nuggets, scales, and rounded or irregular grains; its color is steel-gray. The specific gravity of the crude platinum varies from 14 to 19. The percentage of the metal varies also within wide limits, usually from 70 to 85 per cent. Platinum is almost wholly produced in California and Oregon, and the output for the United States is practically limited to these states.

Owing to its high melting point and great resistance to acids, platinum is extensively used for laboratory utensils. Platinum salts are employed in chemical analysis. In the manufacture of sulphuric acid the metal has been used in making large concentration kettles, but of late gold has been substituted for it. In photography, dentistry, and electric installation much platinum is used. Of late the manufacture of jewelry has consumed large quantities of it. It is extensively used for chains and for the setting of diamonds, the claim being made not only that it is more resistant than silver and harder than gold but that the stones are better offset by platinum and appear larger than in any other kind of setting.

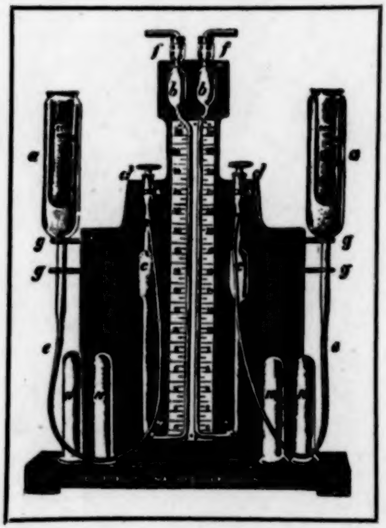
Owing to the high price demanded for platinum during the last two years, a great demand for a substitute has arisen. At one time much platinum was used in the manufacture of incandescent lamps, but is now almost entirely replaced by tungsten. Platinum triangles, used extensively in laboratories, have recently been successfully replaced by similar appliances made of an alloy of nickel and chromium. Nevertheless there remains so many industrial applications of platinum for which no substitutes can be found that it is not likely that the price will be much cheapened in the future.

The present extensive use of platinum in the manufacture of jewelry is stated to be unfortunate, since other metals can be substituted for it, and this fad is undoubtedly one of the principal causes of the great increase in the price of platinum.

BOOKS RECEIVED.

Outlines of Physiology. By Edward G. Jones and Allen H. Bunce, Atlanta School of Medicine. Pages xvi+372. 13x19 cm. Cloth, 1912. $1.50 net. P. Blakiston's Son & Co., Philadelphia.

Mineral Science. By Miner H. Paddock, Technical High School, Providence, R. I.. Pages xiv+148. 13x18 cm. Cloth, 1911. 65 cents. Benj. H. Sanborn & Co., Boston.



BOOK REVIEWS.

Elementary Arithmetic. Pages 388+xxii. 14x19 cm. *Advanced Arithmetic.* Pages 458+xxvii. 14x19 cm. *By Fletcher Durrell, Head of the Mathematical Department of the Lawrenceville School, Lawrenceville, N. J., and Elizabeth Hall, Supervisor of Primary Schools, Minneapolis, Minn.* 1912. Charles E. Merrill Company, N. Y.

The *Elementary Arithmetic* covers the work of the first five grades, and aims not only to develop the principles by methods sound in logic and easily understood by children, but also to give the subject a connection with practical affairs. Throughout the book there is a genuine appeal to the child's own interests. This is shown especially in the industry reviews at the end of each chapter which include the following topics: The Children's Picnic; Arbor Day; Raising Poultry; A Boy's Garden; Keeping a Refreshment Stand; Farm Animals; The Telephone and Telegraph; Post Offices; General Farming; Making a Home.

There are many special drills which in the first grades take the form of games. At the end of the Teachers' Edition the notes to teachers, covering thirty-five pages, give many excellent suggestions for arousing and holding the interest of the children, developing new principles, rapid review and drill work, and so on.

The *Advanced Arithmetic* covers the ground of the last three grades. Though the systematic development of principles is the primary aim, the concrete applications of arithmetic are emphasized by industry reviews treating of manufacture, agriculture, mining and forestry and problems based on facts in geography, history and physics. The large number of problems based on such diverse interests enable the teacher to select those which are of the most importance in the community where the book is being used.

These arithmetics are also published in a Three Book Series. They are well printed with many illustrations and diagrams and contain many features which will appeal to teachers. H. E. C.

Syllabus of Mathematics. Compiled by the Committee on the Teaching of Mathematics to Students of Engineering. Pages 136. 15x23 cm. 1912. 75 cents, prepaid. Professor Henry H. Norris, Cornell University, Ithaca, N. Y.

This committee was appointed at the joint meeting of mathematicians and engineers held in Chicago, December, 1907, under the auspices of the Chicago section of the American Mathematical Society and Sections A and D of the American Association for the Advancement of Science. On the suggestion of officers of the Society for Promotion of Engineering Education who were then present, the committee was instructed to report to this society.

The present volume contains the report presented to the society in June, 1911, and the discussion which followed. It is a synopsis of those fundamental principles and methods of mathematics which, in the opinion of the committee, should constitute the minimum mathematical equipment of the student of engineering. In the five parts the following syllibi are presented: (1) Elementary Algebra, (2) Elementary Geometry and mensuration, (3) Plane Trigonometry, (4) Analytic Geometry, (5) Differential and Integral Calculus. They are intended to include "those fundamental principles which the student ought to have made so completely a permanent part of his mental equipment that he will never need to 'look them up in a book.'" Students and teachers will find many valuable suggestions in this volume. H. E. C.

The Calculus, by Ellery W. Davis, Professor of Mathematics, University of Nebraska, and William C. Brenke, Associate Professor of Mathematics, University of Nebraska. Edited by Earle R. Hedrick. Pages xx+384+61. 13x19 cm. 1912. The MacMillan Company, New York.

A brief *résumé* of the preface will indicate in part the point of view of the authors. As many and as varied applications of the calculus are presented as is possible without venturing into technical fields whose subject-matter is unknown and incomprehensible to the student. Topics are treated with the purpose of bringing out their essential usefulness. Rigorous proofs beyond the present grasp of the student are not given, though gross errors and actual misstatements are avoided. Tradition has not been followed in the omission and inclusion of theorems. The exercises aim to bring out the usefulness of the subject and to give tangible concrete meaning to the concepts involved. An effort is made to remove technical difficulties by the intelligent use of tables. The book aims to give the student a thorough training in mathematical reasoning, to create in him a sure mathematical imagination, and to meet fairly the reasonable demand for enlivening and enriching the subject through applications at the expense of purely formal work that contains no essential principles.

A careful reading of this volume will show that the authors have accomplished their purposes, and have produced a text-book which will undoubtedly find a place in many colleges and technical schools. In the first four chapters the development of the meaning and application of derivatives is based on a study of the slope of curves. This makes a close connection with the student's knowledge of analytic geometry and leads easily to time-rates and related rates and their application in many practical problems.

In chapter V integration is considered first as a reversal of rates and then as a process of summation. Simple algebraic functions only are treated here, and applications are made in finding lengths of curves, areas, and volumes.

In chapter VI the differentiation of transcendental functions is presented and applied in many practical problems. Chapter VII deals with the technique of integration.

In the remaining chapters one finds interesting discussions of Empirical curves, approximate integration, series, partial derivatives, geometry of space, differential equations and so on, with many good problems. A large amount of useful data is put in a compact and unique form in the fifty-eight pages of tables. H. E. C.

The Mammals of Illinois and Wisconsin, by Charles B. Cory, Curator of Department of Zoölogy, Field Museum of National History. Publication 153, Zoölogical Series, Vol. XI. Published by the Museum. Pp. 505. 8vo. 1912.

This volume by Mr. Cory covers all the mammals known to occur in the states of Wisconsin and Illinois, 94 in number. There are numerous keys to orders, families, genera and species. The book is illustrated with numerous half-tones, drawings and cuts, intended either to explain the text or assist in identification of species. There are maps showing the distribution of all the more important species.

While this is a book on strictly scientific lines, it is written in a non-technical style so far as possible so that it may be used by others than specialists. It should prove very useful in schools as a reference book. There are introductory chapters on the anatomy of mammals, taxonomy and classification, classified lists of mammals, estimating age, etc., which

make the book more useful for school work. There are also references to other mammals found in Eastern North America thus widening its scope greatly.

There is nothing to indicate how copies may be obtained, but doubtless such information may be had by applying to the director of the museum. W. W.

An Inductive Chemistry, by R. H. Bradbury. 415 pages 13x20 cm. 1912. D. Appleton & Co., New York.

This book has many new and novel features. It is divided into five books.

Book 1. Sulphur and Its Compounds with Familiar Metals; Familiar Metals and Carbon Which Occur in the Free State in Nature; The Atmosphere, Kinetic Theory of Matter.

Book 2. Compounds of Oxygen with Metals and Non-metals. This part contains five chapters.

Book 3. The Atomic Theory; Important Compounds Containing Hydrogen; four chapters.

Book 4. The Sodium Group; The Chlorine Group; six chapters.

Book 5. Acids containing Oxygen and their Salts; nine chapters.

The subject matter has been written from the standpoint of the student. Much space has been given to practical applications. Modern methods of manufacture are given with illustrations.

Very little space has been given to the chemistry of the household, and sanitation. This is to be regretted.

The book is the product of several years' experience in a large city high school and appears to be a very teachable book. C. M. T.

Laboratory Studies in Chemistry, by R. H. Bradbury. 129 pages. 13x20 cm. 1912. D. Appleton & Co., New York.

This new manual has many good features. The apparatus required is simple and inexpensive. The practical details of the experiments have been worked out with great care, thus requiring a minimum of work and explanation by the instructor.

It contains many new and up-to-date methods. For example, the use of potassium permanganate for generating chlorine, and of formic acid instead of oxalic acid for preparing carbon monoxide.

The book contains 87 well selected experiments of which 15 are quantitative. One of the most valuable features of the book is the classification. Facts which relate to some central topic are not scattered through the book, but are collected into a compact laboratory exercise which serves to make the matter clear once for all.

It is intended to be used with the author's text but can be easily used with any book. C. M. T.

The Teaching of Plane Geometry, by H. Y. Benedict, Professor of Applied Mathematics, and J. W. Calhoun, Instructor in Pure Mathematics, University of Texas. 57 pages. 15x23 cm. Paper. 1912. The University of Texas, Austin, Texas.

This Bulletin, No. 243, may be obtained free of charge upon application to the University of Texas. Its aim is to supplement the Report of the Committee of Fifteen of the N. E. A., with special reference to the needs of the high schools of Texas. Some of the topics are: uses of geometry, its relation to other subjects, time and place in the curriculum, on the beginning of logical geometry, the theorems of geometry, modes of proof, sample proofs, loci, limits, and approximations.

The authors have displayed a great deal of common sense in the discussion of these topics, and the pamphlet will prove very helpful to all teachers of geometry. Indeed, it can be recommended to those who are intending to write a textbook on this subject. H. E. C.

Plane and Solid Geometry, by C. A. Hart, Wadleigh High School, New York, and D. D. Feldman, Erasmus Hall High School, Brooklyn. Pages, viii +488. 14x19 cm. 1912. American Book Company, New York.

Teachers who wish to use a new text-book without departing from the traditional treatment of elementary geometry will be interested in this book, and they will undoubtedly find it satisfactory. If proofs must be printed in full, the arrangement in parallel form is an excellent device, and the compact and condensed form of statement in the proofs is highly commendable. The proofs of some theorems and constructions are left as exercises for the student. Most of the propositions are followed by several exercises and problems. Among the large number of exercises, 1667 in all, one finds a few "so-called" (the authors' designation) practical problems.

There seems to be always a conscious effort on the part of the authors to proceed with extreme accuracy and logical rigor, and this makes the occasional slips quite noticeable. The proofs of the incommensurable cases are given, and the theorem "If two variables are always equal, etc.," holds its time-honored place. The historical notes give an interesting account of the life and work of many great mathematicians. All of the figures are well drawn, and in the solid geometry the skillful shading of the figures aids greatly in comprehending the proofs. H. E. C.

Owing to the fact that we have changed the office of publication the January issue has been delayed for one week. Hereafter this Journal will be mailed on the 20th of the month preceding the month of publication.